Wavelet transforms and their applications for ITER

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ITER International School
Aix-en-Provence
26° August 2014

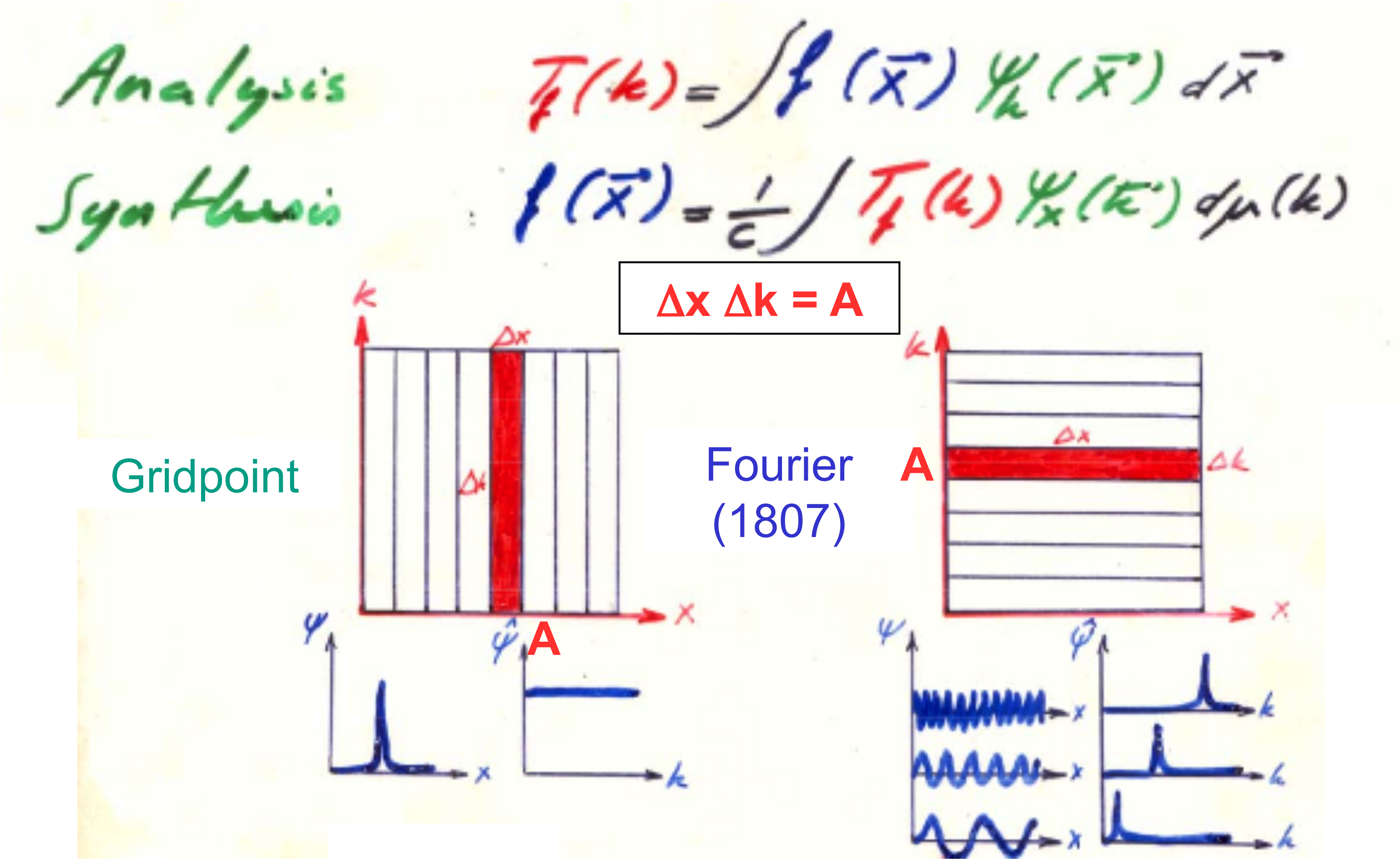
Outline

1. *Wavelet transforms*
   Integral transforms
   Continuous wavelet transform
   Fast orthogonal wavelet transform

2. *Applications*
   Extraction of coherent structures
   out of turbulent flows

3. *Proposals for ITER*
   Analyzing signals measured in the edge plasma
   Denoising fast camera visible light movies
   Improving PIC codes

Integral transforms



Continuous Fourier transform

$$f(x) \in L^1(R) \cap L^2(R)$$

Analysis

$$\widehat{f}(k) = \int_{-\infty}^{\infty} f(x) e^{-i2\pi k \cdot x} dx$$

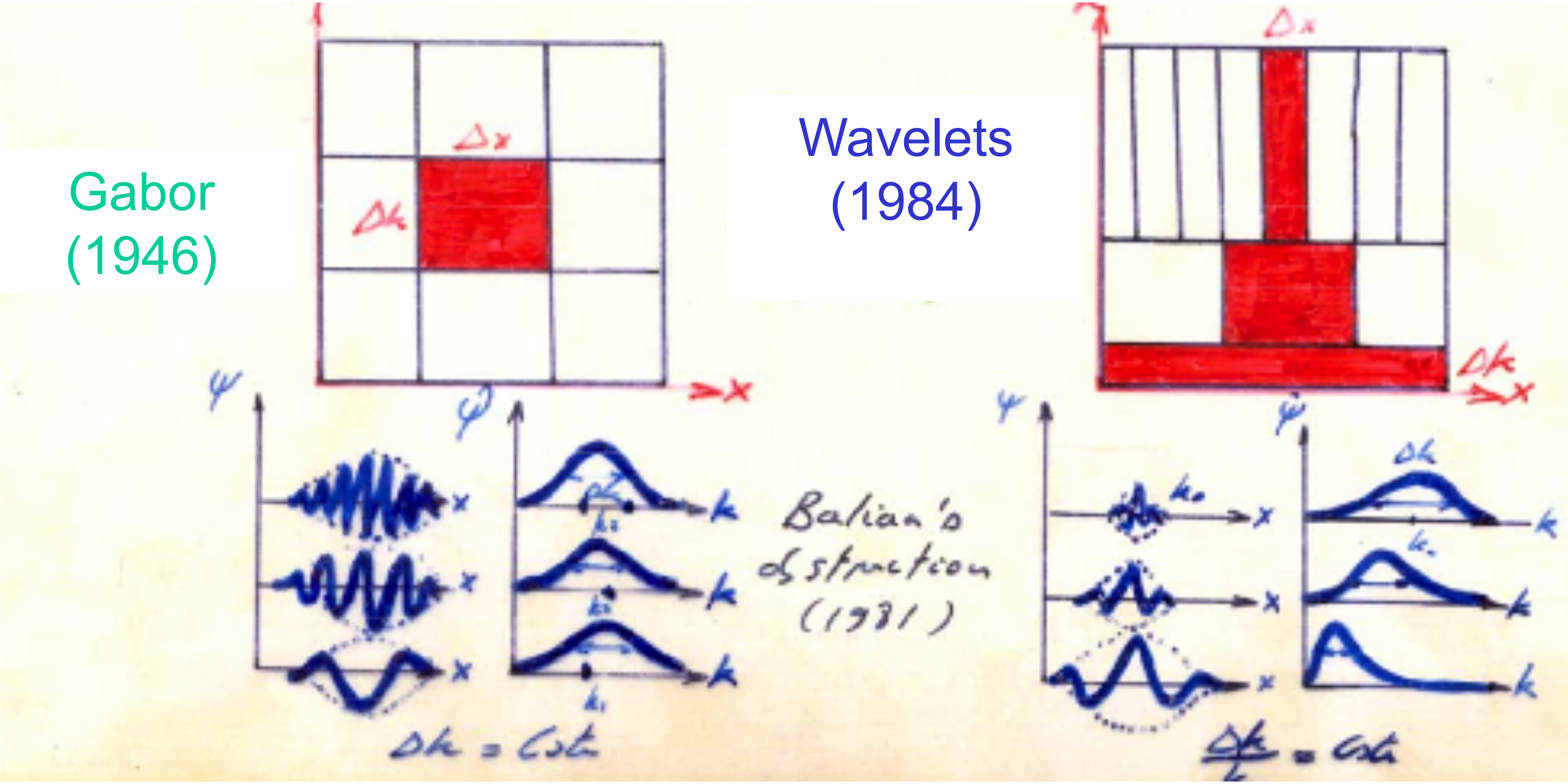
Synthesis

$$f(x) = \int_{-\infty}^{\infty} \widehat{f}(k) e^{i2\pi k \cdot x} dk$$

Parseval's identity

$$\int_{-\infty}^{\infty} f_1(x) \cdot f_2(x) dx = \int_{-\infty}^{\infty} \widehat{f}_1(k) \cdot \widehat{f}_2(-k) dk$$

Optimal phase space tiling



Space-wavenumber representation

$\Delta x \, \Delta k =$ information atom

Space-scale representation

Choice of the analyzing wavelet

Admissibility condition

$$C_\psi = \int_0^\infty \left|\widehat{\psi}(k)\right|^2 \frac{dk}{|k|} < \infty$$

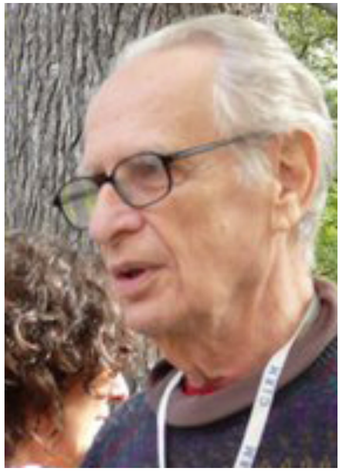
$$\int_{-\infty}^{\infty} \psi(x)\, dx = 0 \qquad \text{or} \quad \widehat{\psi}(0) = 0$$

Jean Morlet

Analyzing wavelet family generated by translation (b) and dilation (a)

$$\psi_{a,b}(x) = \frac{1}{\sqrt{a}}\, \psi\left(\frac{x-b}{a}\right)$$

Alex Grossmann



Grossmann and Morlet,
Decomposition of Hardy functions into square integrable wavelets of constant shape,
SIAM J. math. Anal., ***15****(4), 723-736, 1984*

Continuous wavelet transform (CWT)

Analysis

$$\tilde{f}(a,b) = \int_{-\infty}^{\infty} f(x)\psi_{a,b}^{*}(x)\,dx$$

Synthesis

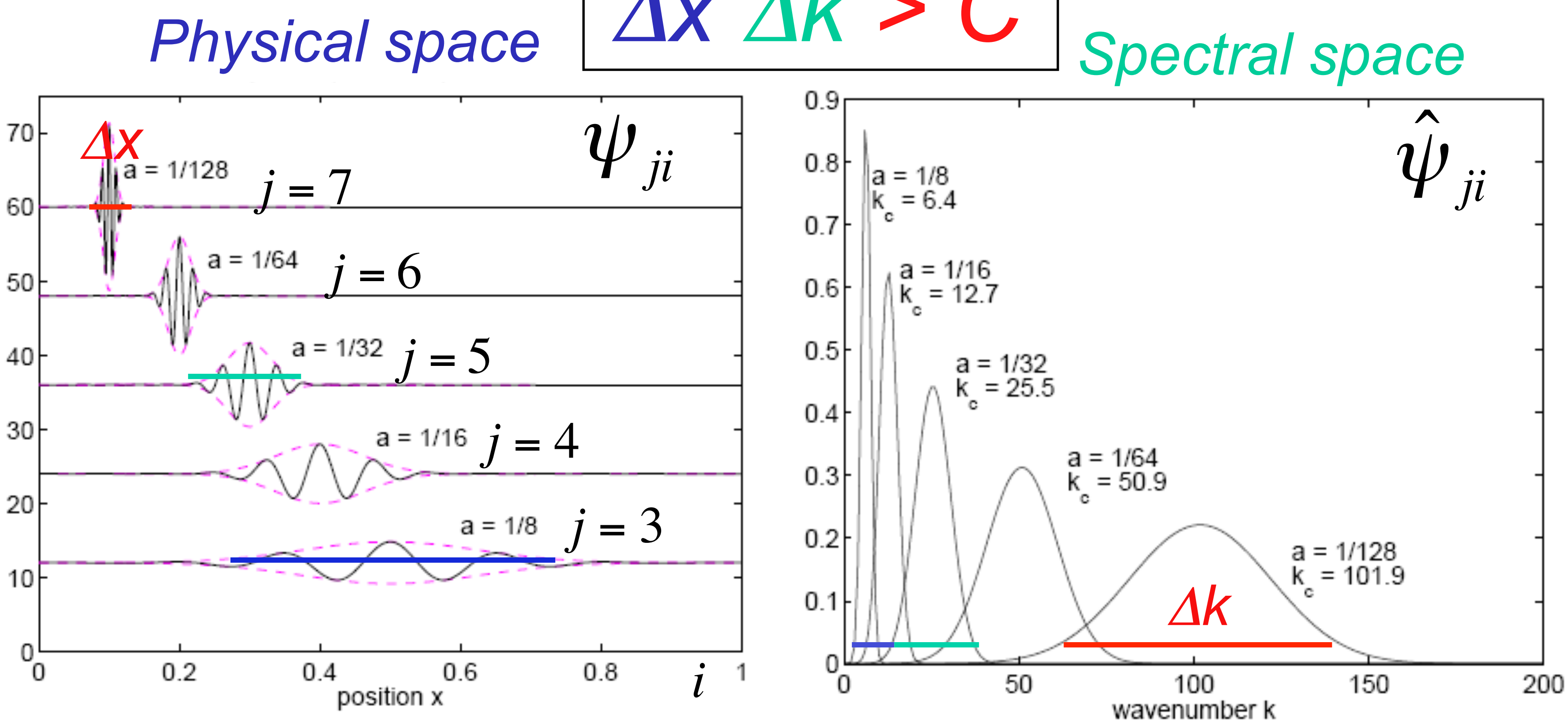
$$f(x) = \frac{1}{C_\psi} \int_{0}^{\infty}\int_{-\infty}^{\infty} \tilde{f}(a,b)\,\psi_{a,b}(x)\,\frac{da\,db}{a^2}$$

Parseval's identity

$$\langle f_1, f_2 \rangle = \int_{-\infty}^{\infty} f_1(x)\,f_2^{*}(x)\,dx = \frac{1}{C_\psi}\int_{0}^{\infty}\int_{-\infty}^{\infty} \tilde{f}_1(a,b)\,\tilde{f}_2^{\,*}(a,b)\,\frac{dadb}{a^2}$$

Wavelet representation

$\Delta x\ \Delta k > C$

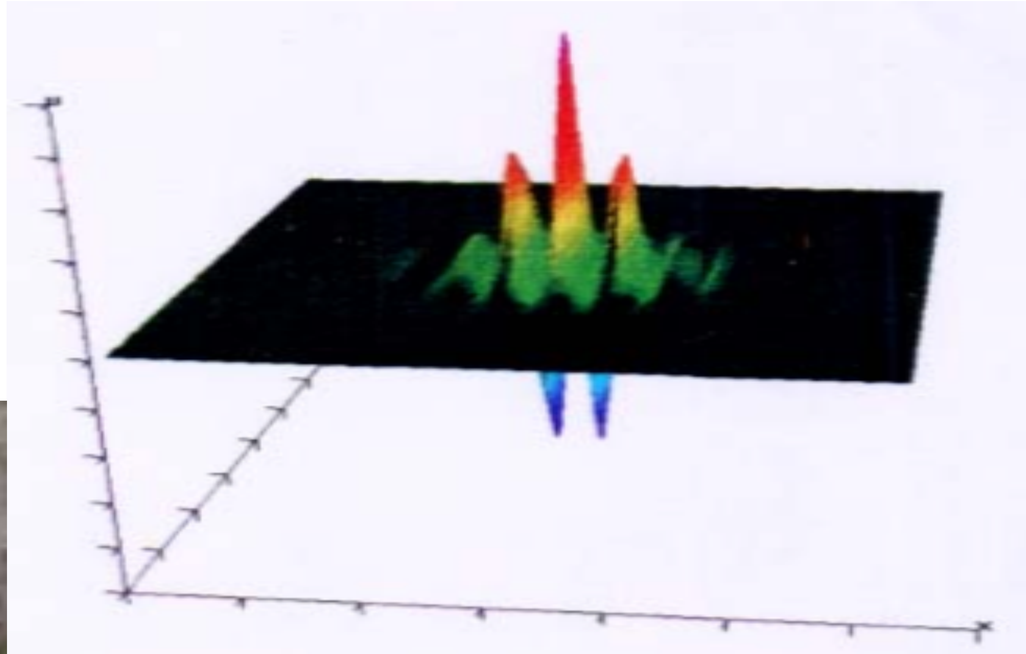


Farge,
Wavelet transforms and
their applications to turbulence
Ann. Rev. Fluid Mech., ***92****, 1992*

Farge and Schneider,
Wavelets: application to turbulence,
Encyclopedia of Mathematical Physics,
Springer, 408-42, 2006

2D continuous wavelet transform

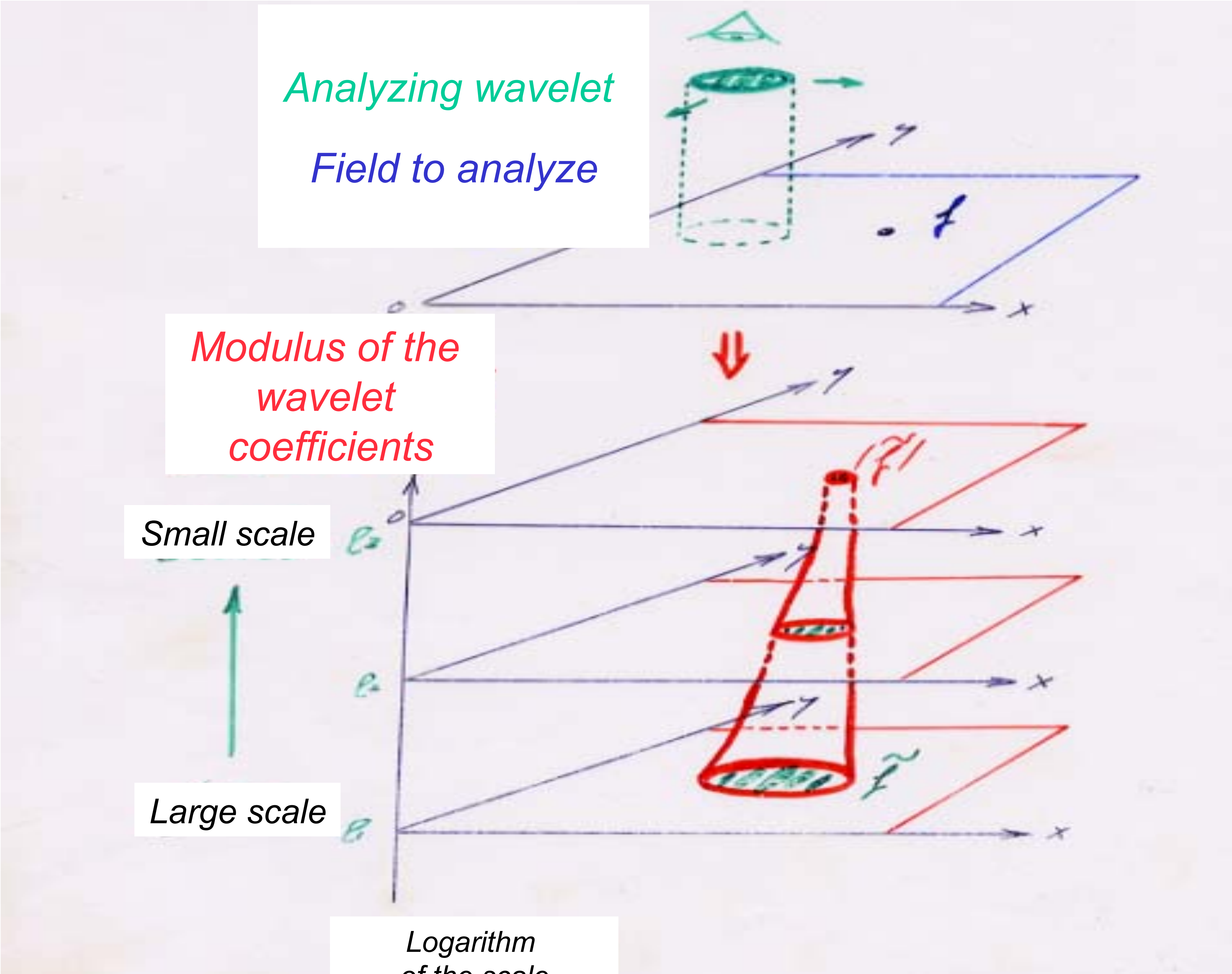
Jean-Pierre Antoine



2D Morlet mother wavelet

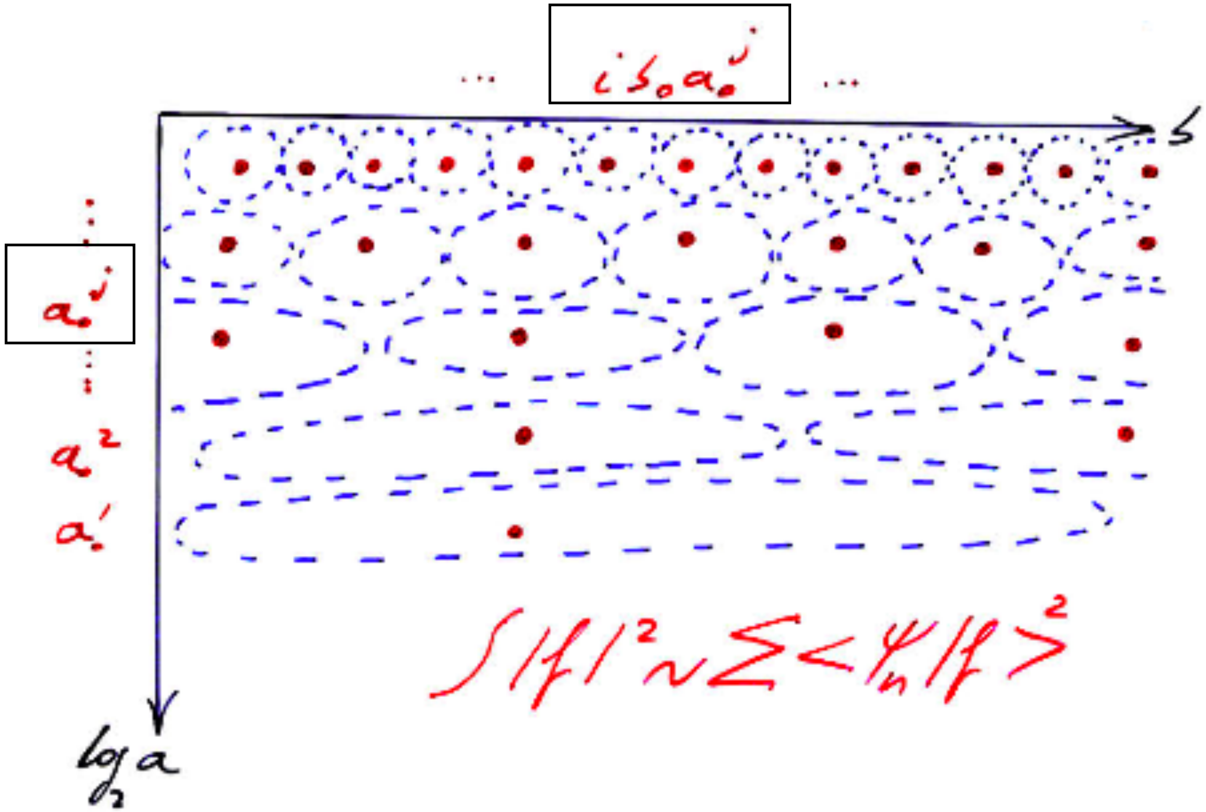
Romain Murenzi

The wavelet family is generated by translating, dilating and rotating the 2D mother wavelet


Analyzing wavelet
Field to analyze
Modulus of the wavelet coefficients
Small scale
Large scale
Logarithm

Wavelet frame

We can then select a finite number of wavelets restricted to a discrete grid optimally chosen such that the wavelet family associated to this grid constitutes a quasi-orthogonal basis $\Longrightarrow$ *a wavelet frame*



For example

for Marr wavelet

we need

$a_0=2^{1/2}$

$b_0=1/2$

Orthogonal wavelet transform

Wavelet analysis :

$$\widetilde{f}_{ji} = \langle \psi_{ji} | f \rangle \quad \text{with} \quad \psi_{ji} = 2^{j/2}\psi(2^j x - i)$$

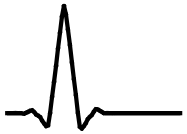
Wavelet synthesis :

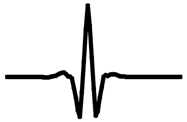
$$f = \sum_{ji} \langle \psi_{ji} | f \rangle \psi_{ji}$$

A signal sampled on N points is
wavelet analyzed and synthetized in CN operations
if one uses compactly-supported wavelets
computed from a quadratic mirror filter of length M.

2D orthogonal wavelets

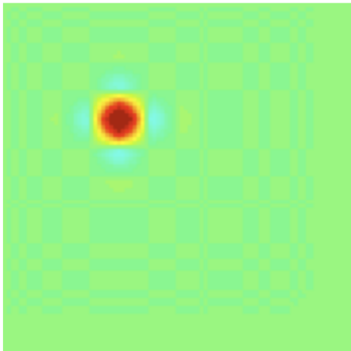
Scaling function *Wavelet*

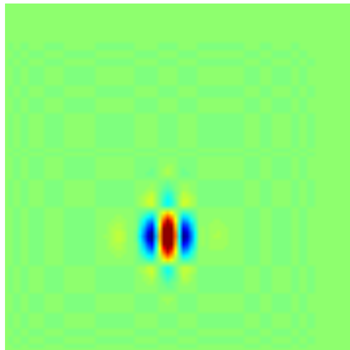
ϕ 

ψ 

$\phi(x)\phi(y)$

Coarse approximation



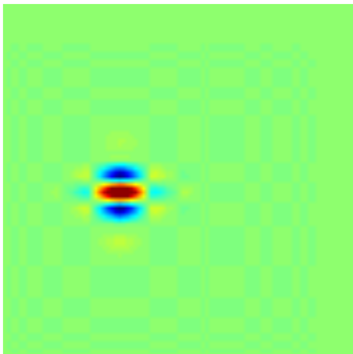


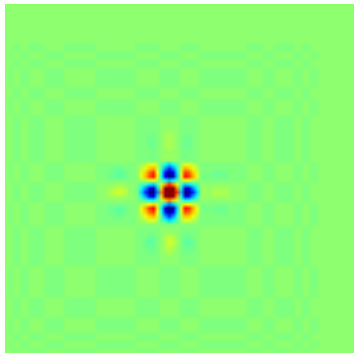
$\psi(x)\phi(y)$

Horizontal details

$\phi(x)\psi(y)$

Vertical details





$\psi(x)\psi(y)$

Diagonal details

3D orthogonal wavelets

A 3D vector field v(x) sampled on $N = 2^{3J}$ equidistant grid points

$\psi_\lambda(\boldsymbol{x})$ 3D wavelet $\longrightarrow$ orthogonal wavelet series

$$\boldsymbol{v}(\boldsymbol{x}) = \sum_{\lambda \in \Lambda} \widetilde{\boldsymbol{v}}_\lambda \psi_\lambda(\boldsymbol{x}), \quad \widetilde{\boldsymbol{v}}_\lambda = \langle \boldsymbol{v}, \psi_\lambda \rangle$$

$$\Lambda = \{\lambda = (j, i_n, \mu \ldots, \ldots, J-1,\ i_n = 0, \ldots, 2^j - 1, n = 1, 2, 3,\ \text{and}\ \mu = 1, \ldots, 7\}$$

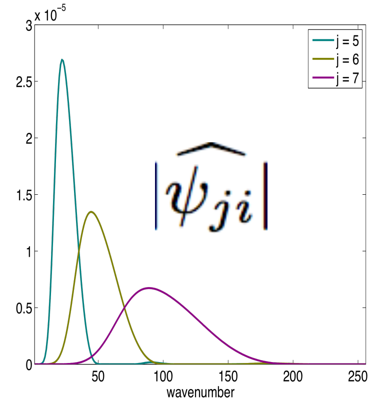
$N_j = 7 \times 2^{3j}$, wavelet coefficients at a scale indexed by j

- fast algorithm with linear complexity
- no redundancy between the coefficients

We use Coifman 12 wavelet
compactly supported with four vanishing moments.

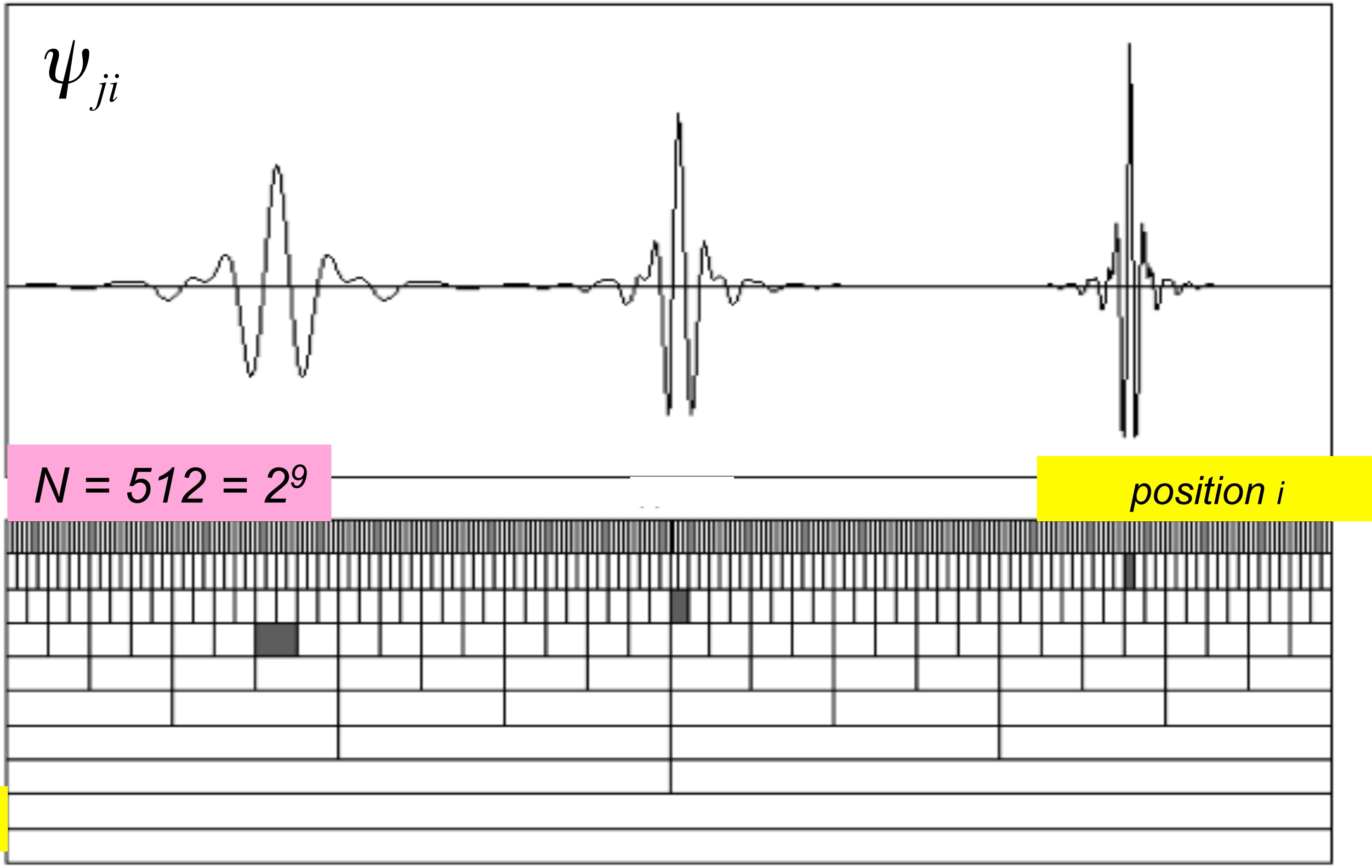
Orthogonal wavelet representation

Wavelets



Wavelet coefficients

$$\tilde{f}_{ji} = \langle \psi_{ji} | f \rangle$$



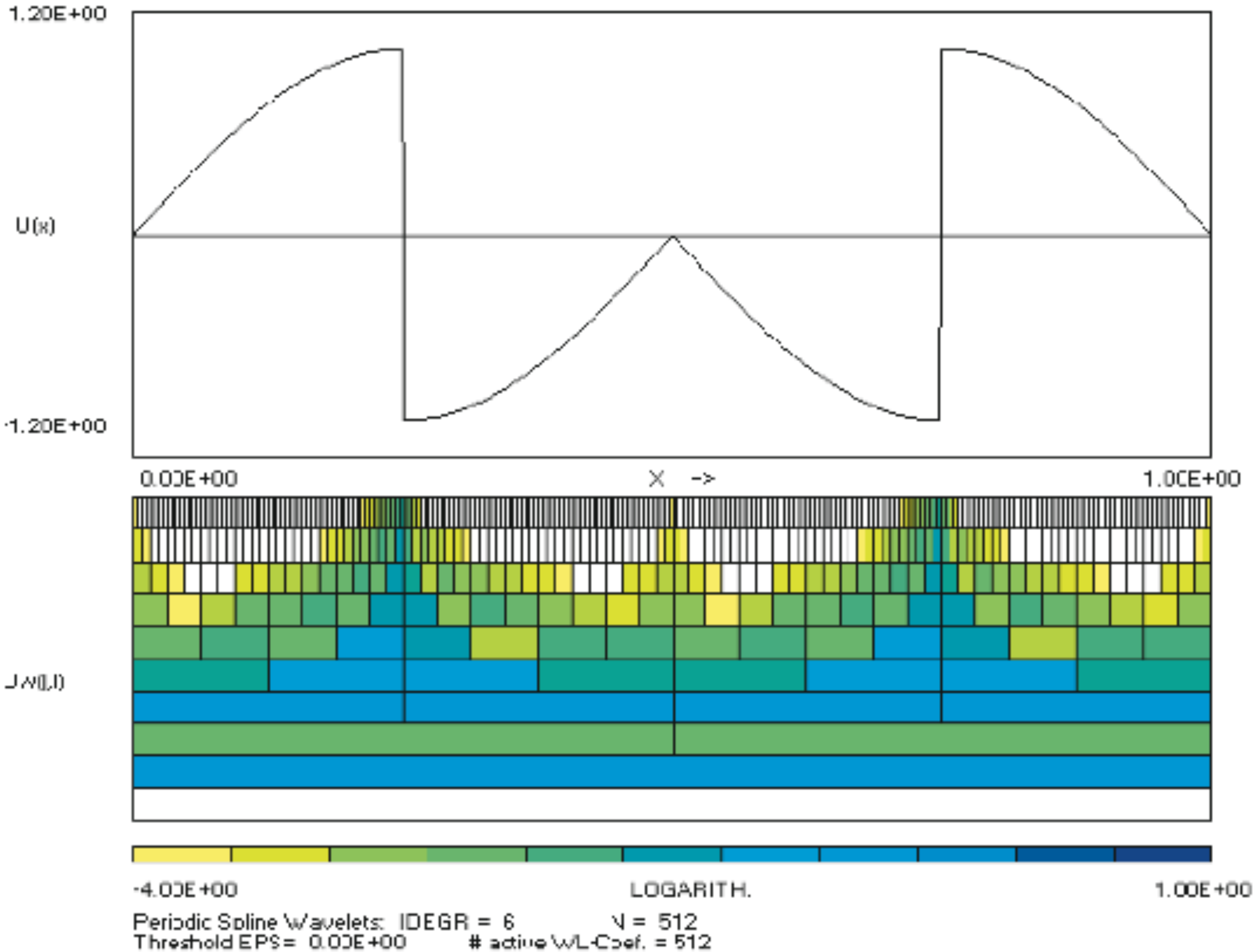
scale j

Mallat,
A wavelet tour of
signal processing, 3rd edition,
Academic Press, 2008

Academic example

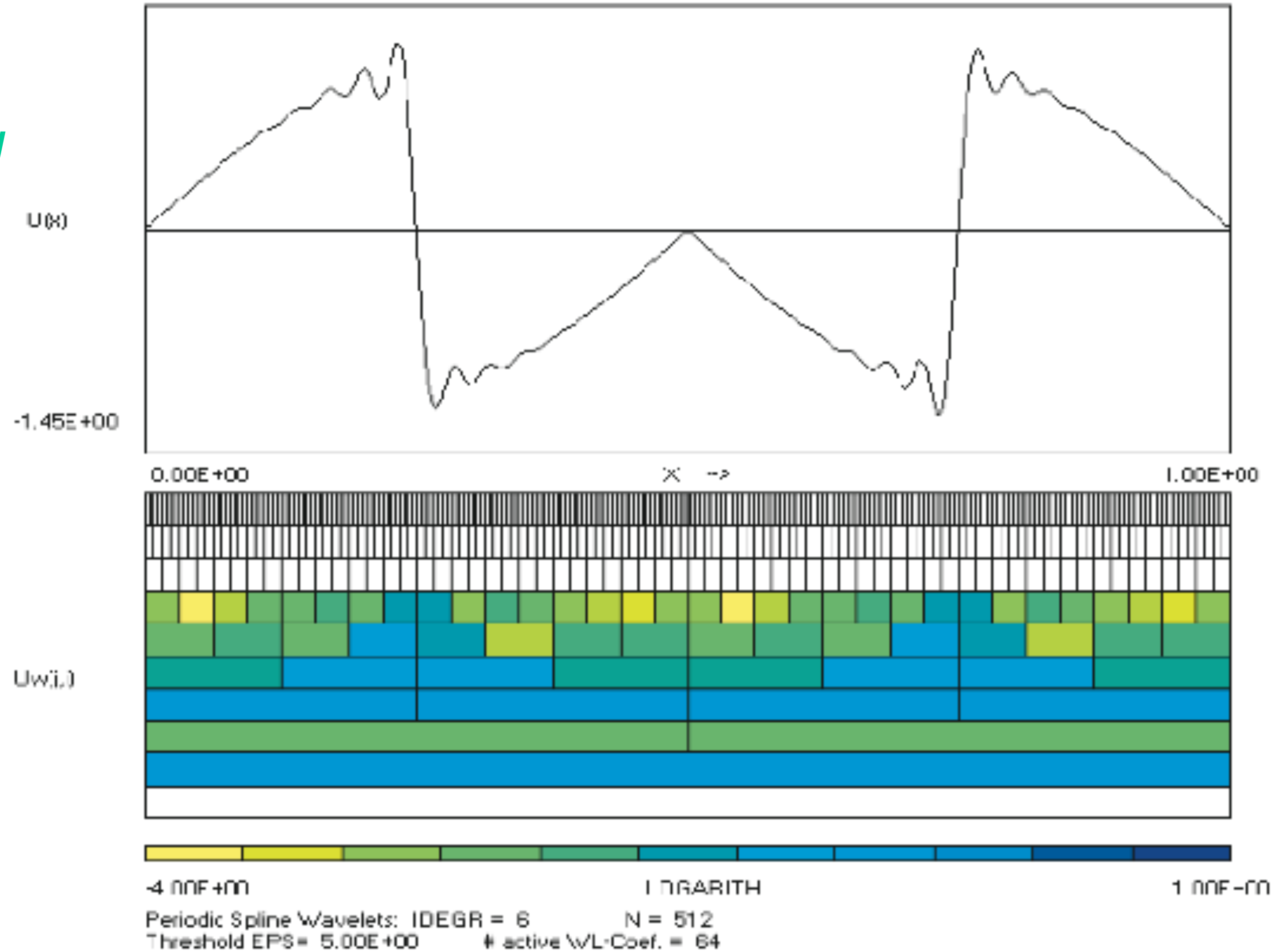
Function sampled on N = 512 grid-points

N = 512 wavelet coefficients



Linear approximation

Function reconstructed from 64 wavelet coefficients



64 wavelet coefficients such that

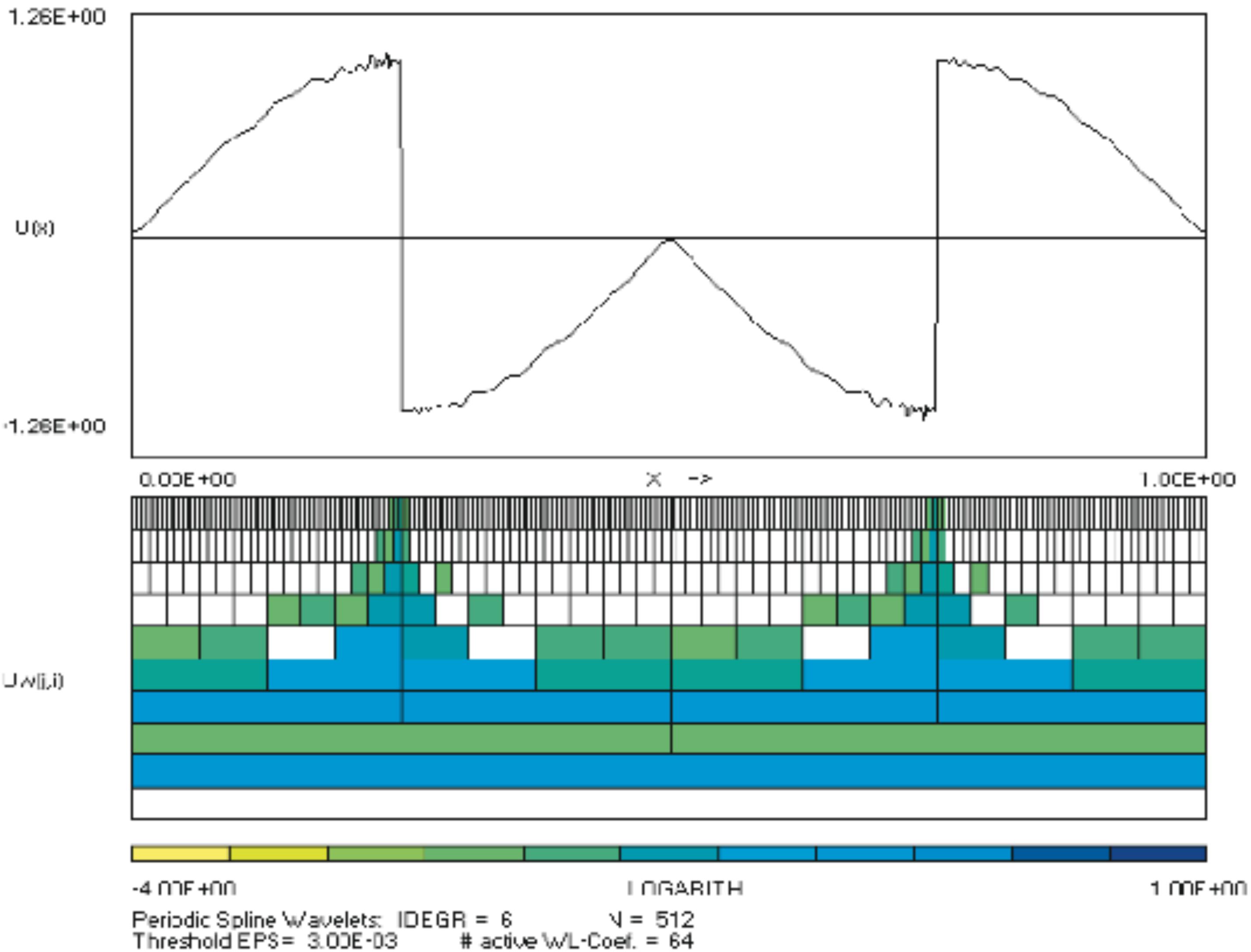
$$j \leq 6$$

Nonlinear approximation

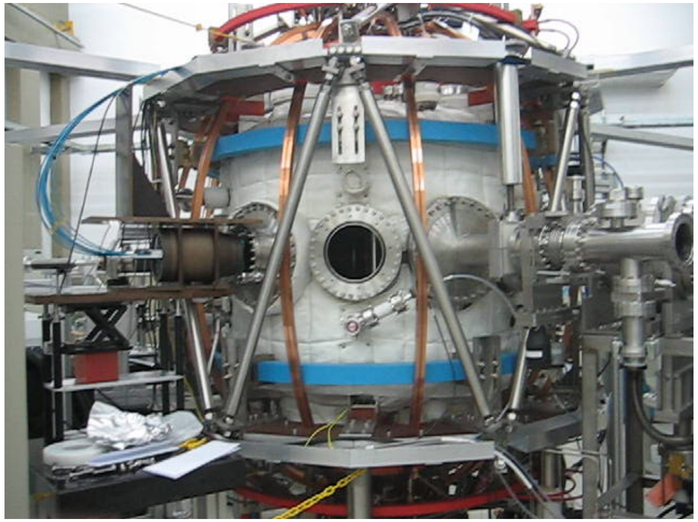
Function reconstructed from 64 wavelet coefficients

64 wavelet coefficients such that

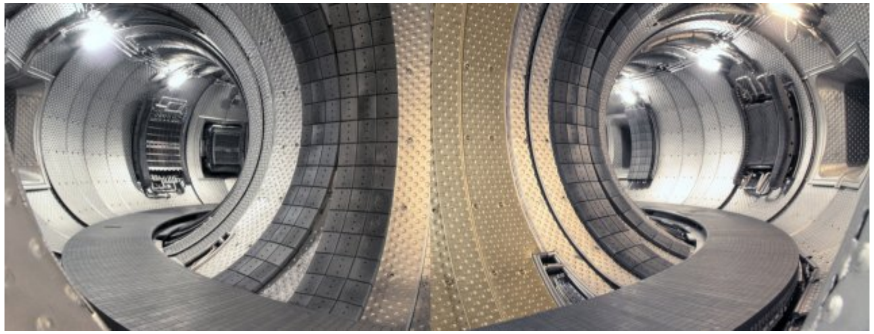
$$|\widetilde{f}_{ji}| > \epsilon$$



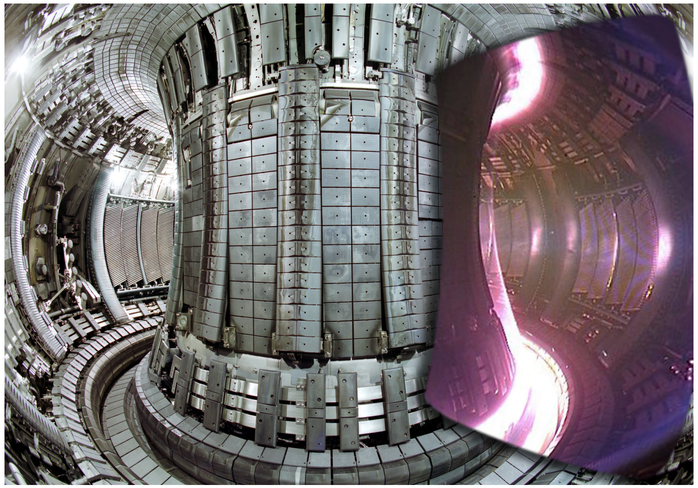
Application to tokamaks



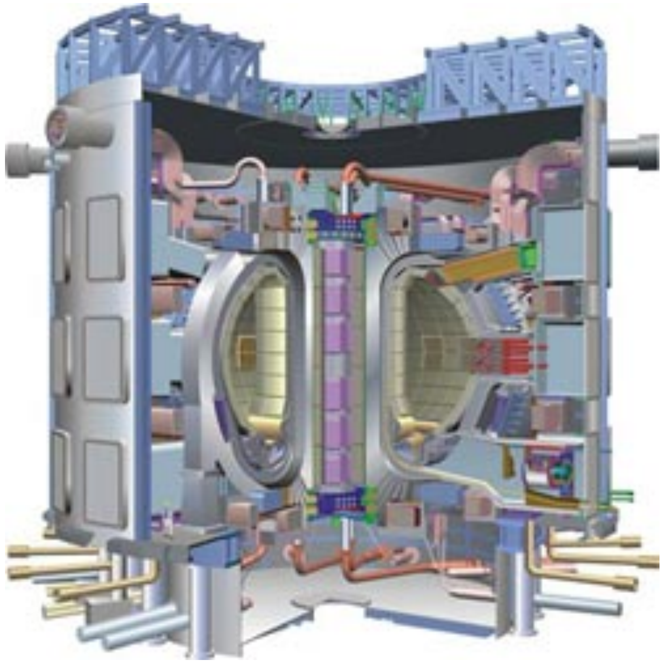
ETE, INPE (Brazil)



Tore-Supra, Cadarache (France)



JET, Culham (Europe)



ITER (World)

Turbulent edge plasma in the SOL

Turbulent edge plasma
in the SOL (Scrape Off Layer),
where there are very large density
and temperature gradients

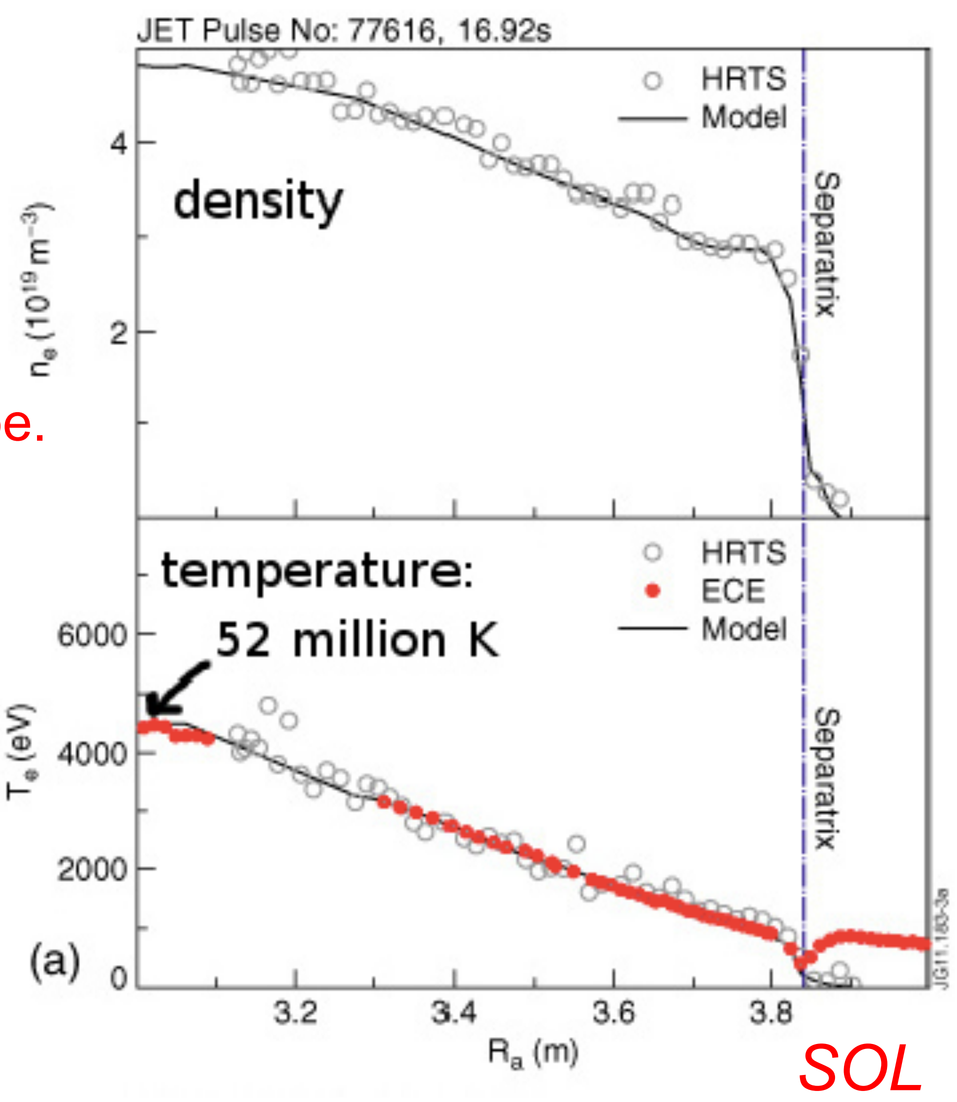
⟹

Ion density fluctuations measured
by a fast reciprocating Langmuir probe.

Edge plasma is colder
than core plasma, then ions
and electrons can recombine.
Recombination and
later desexcitation induce
visible light emission
(*e.g.*, Hα line with λ=656 nm).

⟹

Video acquisition using
a fast camera (40 KHz).



How to extract coherent structures?

Since there is not yet a universal definition of coherent structures
which emerge out of turbulent fluctuations due to the nonlinear interactions,
we adopt an apophetic method :
instead of defining what they are, we define what they are not.

For this we propose the minimal statement :
'Coherent structures are not noise'

$\Longrightarrow$

Extracting coherent structures becomes a denoising problem,
not requiring any hypotheses on the structures themselves
but only on the noise to be eliminated.

Choosing the simplest hypothesis as a first guess,
we suppose we want to eliminate an additive Gaussian white noise,
and for this we use a nonlinear wavelet filtering.

Farge, Schneider et al., Phys. Fluids, ***15*** *(10), 2003*

Azzalini, Farge, Schneider, ACHA, ***18*** *(2), 2005*

Denoising using wavelets

Gaussian **white noise** is by definition **equidistributed** among all modes and the amplitude of the coefficients is given by its r.m.s., whatever the functional basis one considers.

Therefore the coefficients of a noisy signal whose amplitudes are larger than the r.m.s. of the noise belong to the denoised signal. This procedure corresponds to **nonlinear filtering**.

The advantage of performing such a nonlinear filtering using the wavelet representation is that the **wavelet coefficients** preserve the space locality, since wavelets are functions localized in both physical and spectral space.

Since we do not know *a priori* the r.m.s. of the noise, we have proposed an iterative procedure which takes as first guess the r.m.s. of the noisy signal.

Azzalini, M. F., Schneider, 2005
Appl. Comput. Harmonic Analysis, ***18*** *(2)*

Wavelet denoising algorithm

Apophatic method :

- no hypothesis on the structures,
- only hypothesis on the noise,
- simplest hypothesis as our first choice.

Hypothesis on the noise :

$$f_n = f_d + n$$

n	*Gaussian white noise,*
$\langle f_n^2 \rangle$	*variance of the noisy signal,*
N	*number of coefficients of* f_n.

Wavelet decomposition :

$$\tilde{f}_{ji} = \langle f \mid \psi_{ji} \rangle$$

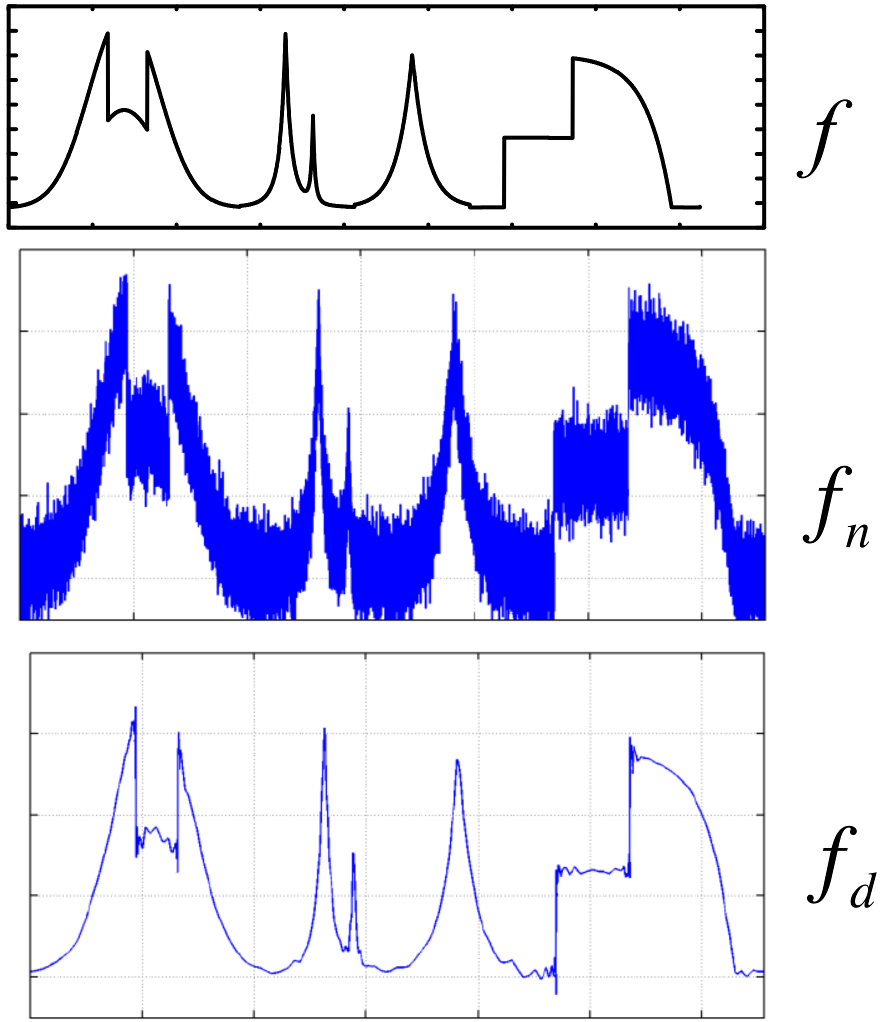
j scale,
i position

Estimation of the threshold :

$$\varepsilon_n = \sqrt{2 \langle f_n^2 \rangle \ln(N)}$$

Wavelet reconstruction :

$$f_d = \sum_{ji : |\tilde{f}_{ji}| > \varepsilon_n} \tilde{f}_{ji} \psi_{ji}$$



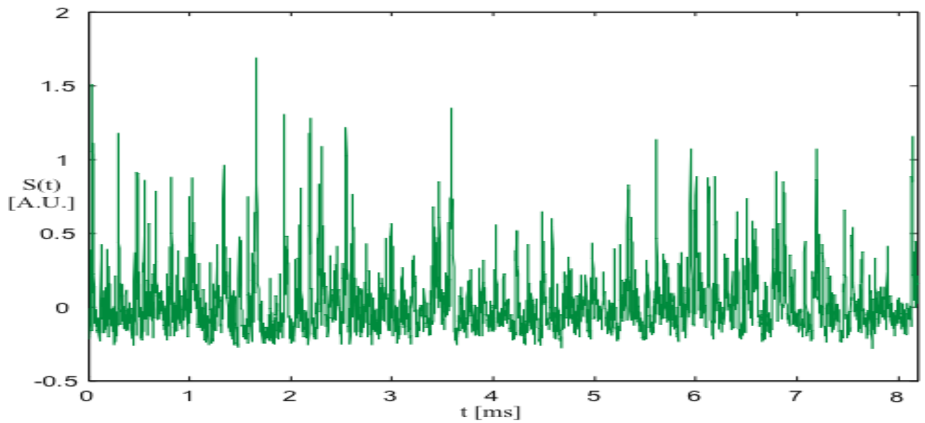
Donoho, Johnstone, Biometrika, ***81****, 1994*

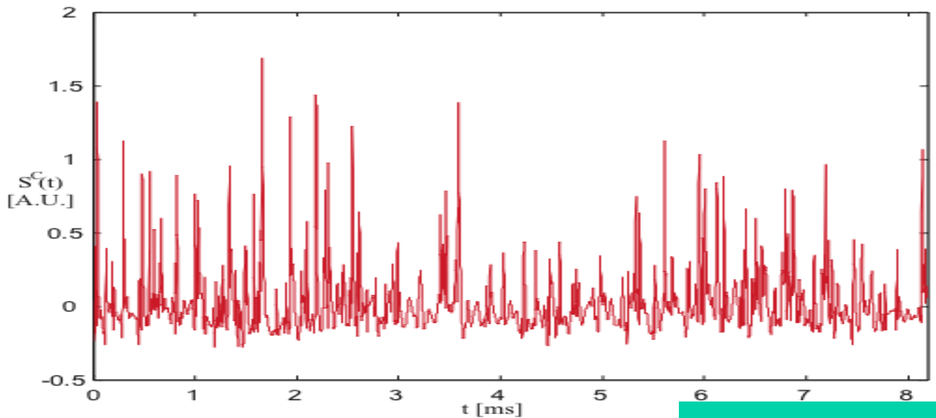
Azzalini, M. F., Schneider, ACHA, ***18*** *(2), 2005*

Extraction of coherent structures SOL

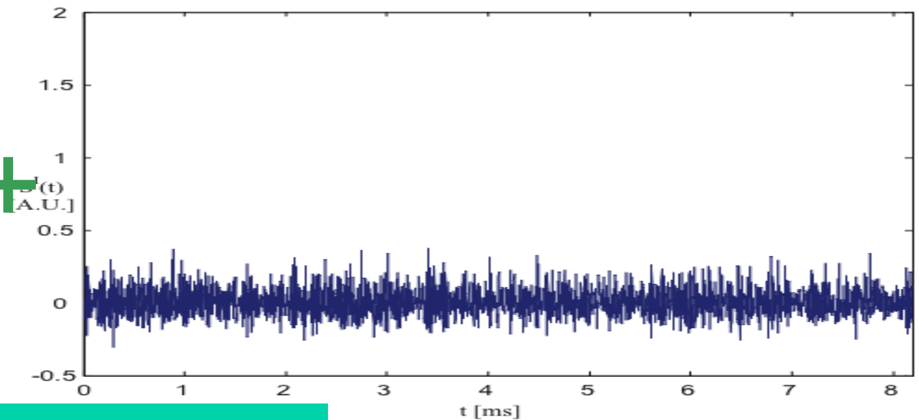
Ion density fluctuations measured by a fast reciprocating Langmuir probe in the SOL of the tokamak Tore Supra

(Pascal Devynck, Tore-Supra, CEA-Cadarache)





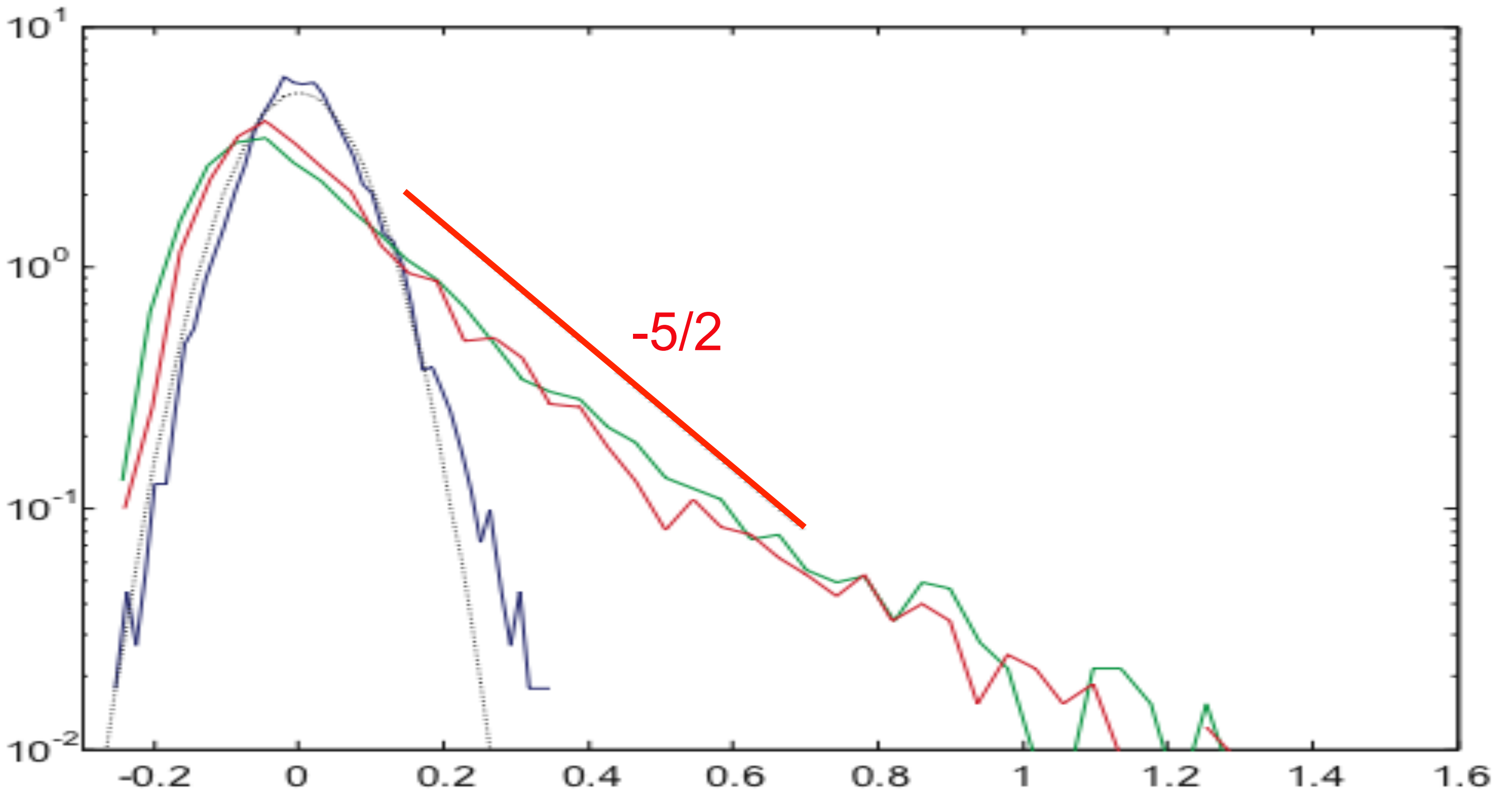
Coherent



Incoherent

Farge, Schneider & Devynck
Phys. Plasmas, ***13****, 042304, 2006*

PDF of the density fluctuations

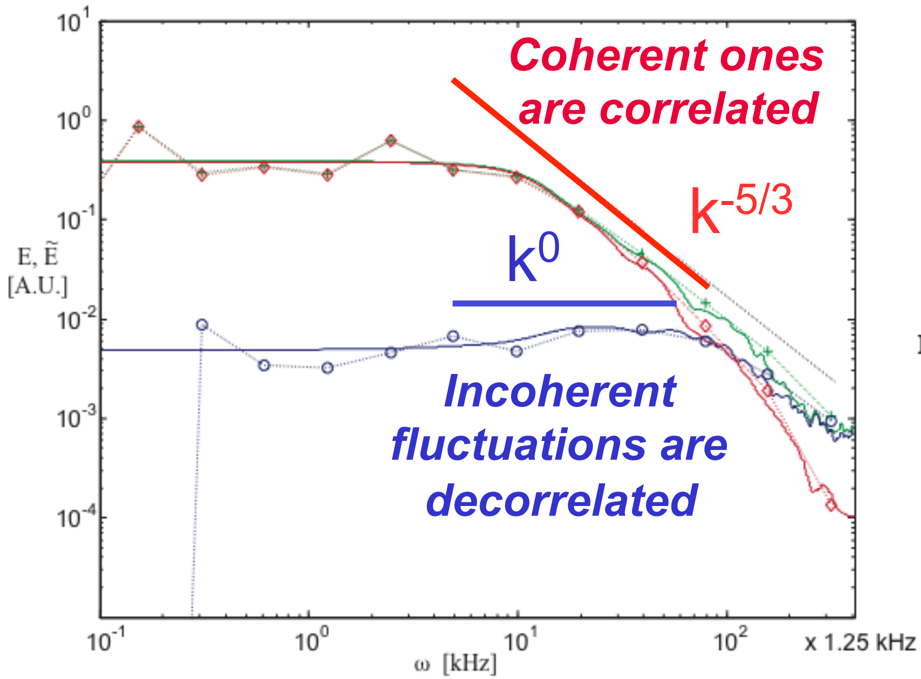


Total fluctuations = coherent + incoherent fluctuations

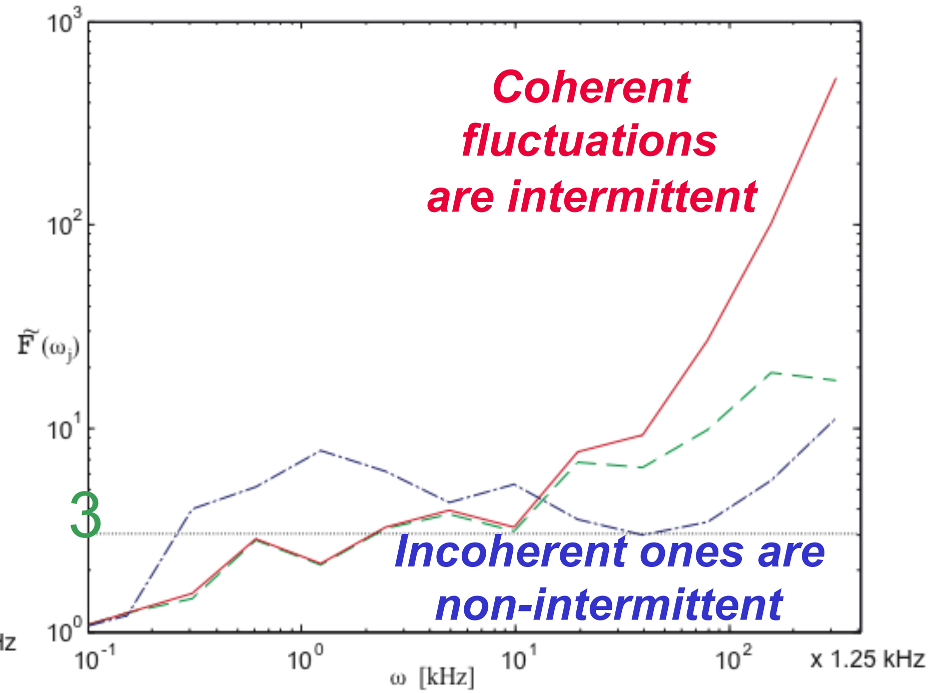
Farge, Schneider & Devynck, Phys. Plasmas, ***13****, 2006*

Correlation and intermittency

Scalogram
(stabilized periodogram)



Flatness versus scale
(from wavelet coefficients)



Total fluctuations = coherent + incoherent fluctuations

Farge, Schneider & Devynck, Phys. Plasmas, ***13****, 2006*

Fast visible light camera

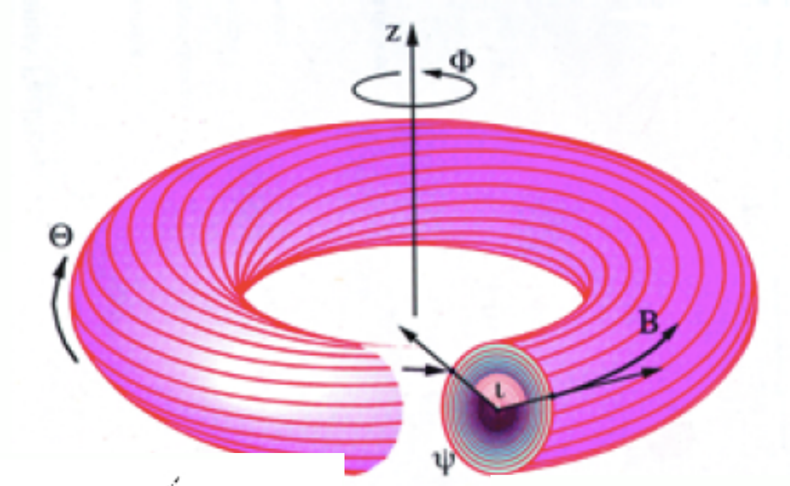
A fast camera from the Nancy team *(G. Bonhomme and F. Brochard)*
was installed on Tore-Supra *(N. Fedorczak and P. Monier-Garbet)*.

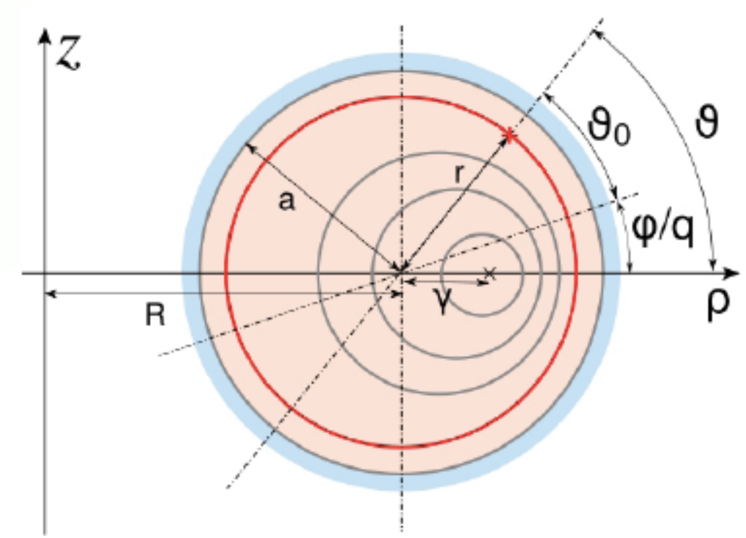
An helical Abel transform relates the plasma light emissivity S
to the integral of the volume emissivity received by the camera I=KS,
where K is a compact continuous operator.
Reconstruction of S from I is an inverse problem
which becomes very difficult
when S is corrupted by noise,
then solving K^{-1} is an ill-posed problem.

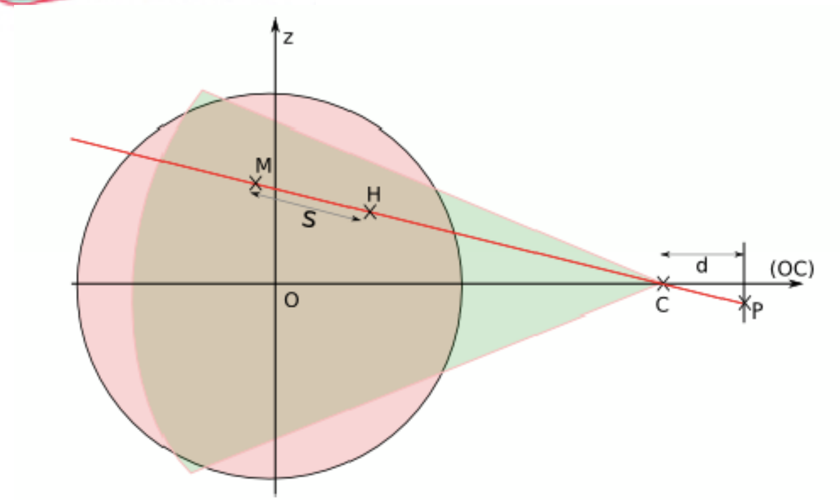
$\Longrightarrow$

Tomographic inversion
using wavelet-vaguelette decomposition
as an alternative to SVD
(Singular Value Decomposition).

Image tomography





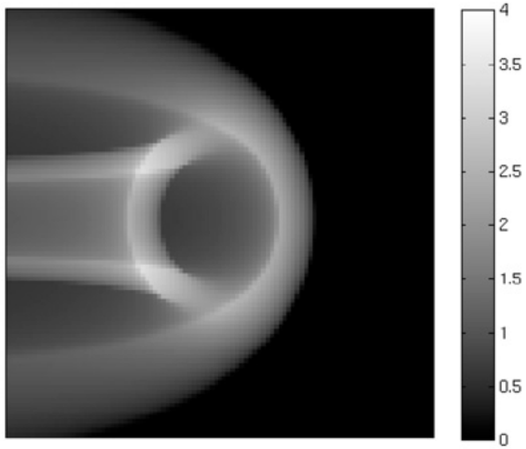


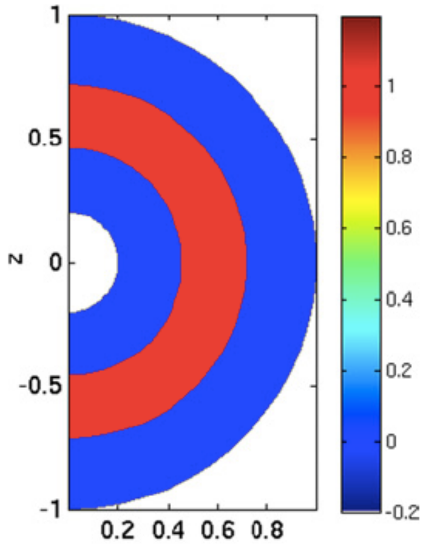
$$I_0(x, y) = \int_{s_C}^{\infty} S_0(\Psi(s), \theta(s), \varphi(s))\, \mathrm{d}s$$

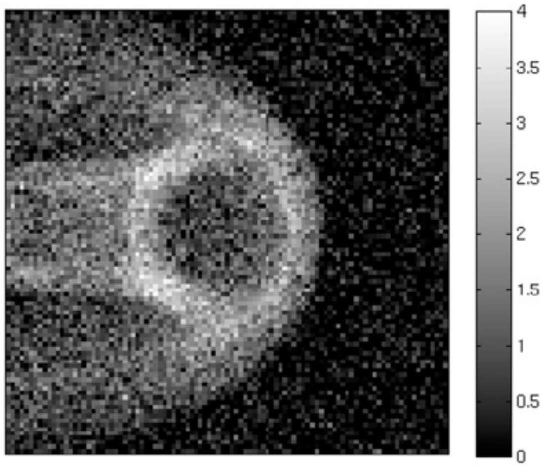
Tomography inversion in presence of noise

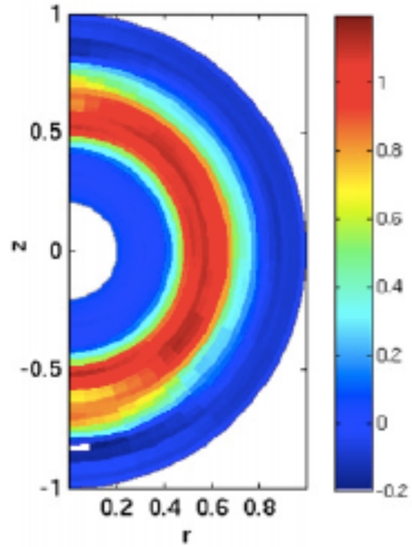
Image received by the camera: integral of the volume emissivity I=KS

Plasma light emissivity S



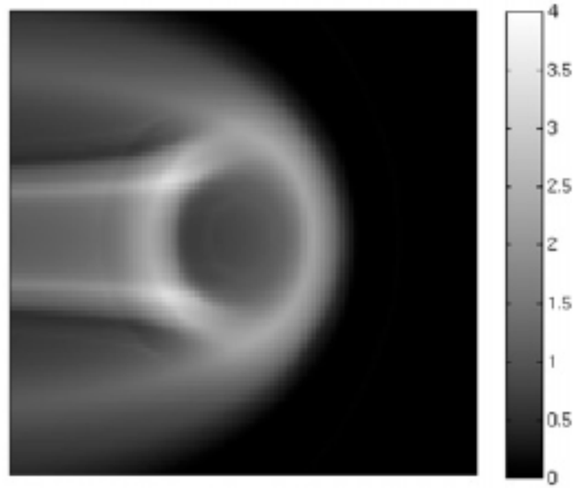




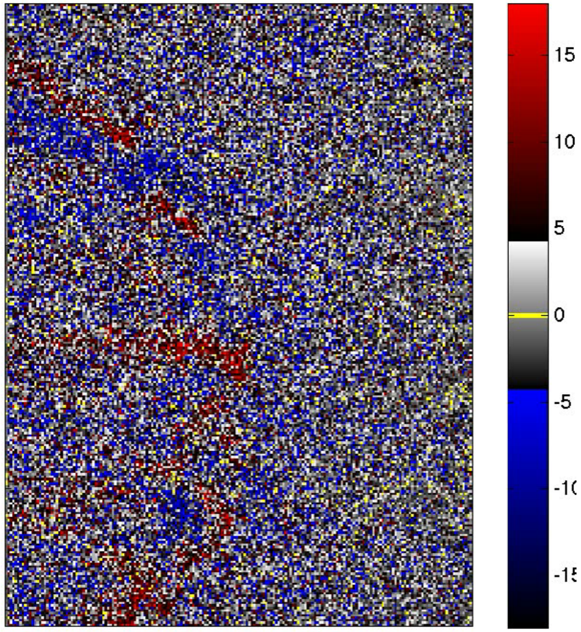


Nguyen, Fedorczak, Brochard, Bonhomne, Schneider, Farge, Monier-Garbet, Nuclear Fusion, ***52****, 2012*

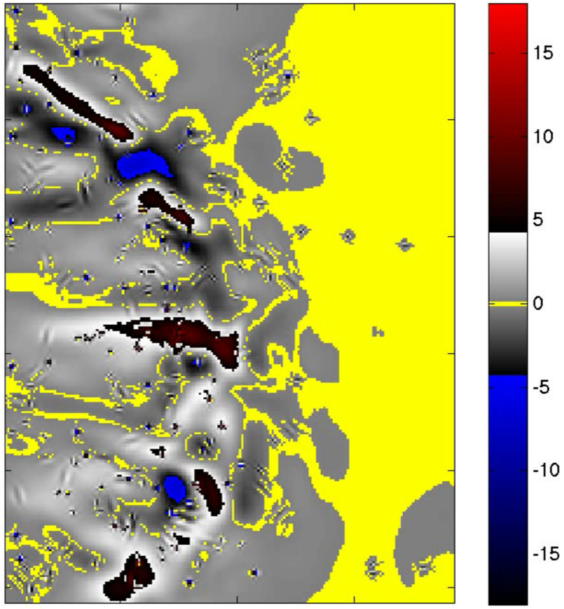
Denoised plasma emissivity



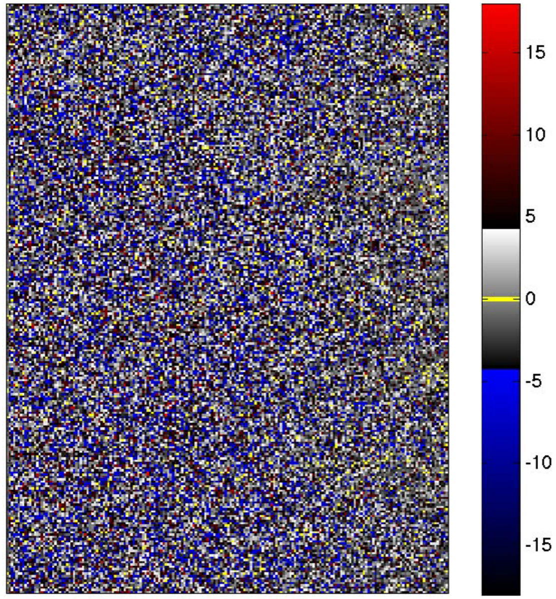
Movie from a fast camera in Tore-Supra tokamak



Noisy images



Coherent structures



Incoherent background

*Nguyen, Fedorczak, Brochard, Bonhomne, Schneider, Farge, Monier-Garbet, Nuclear Fusion, **52**, 2012*

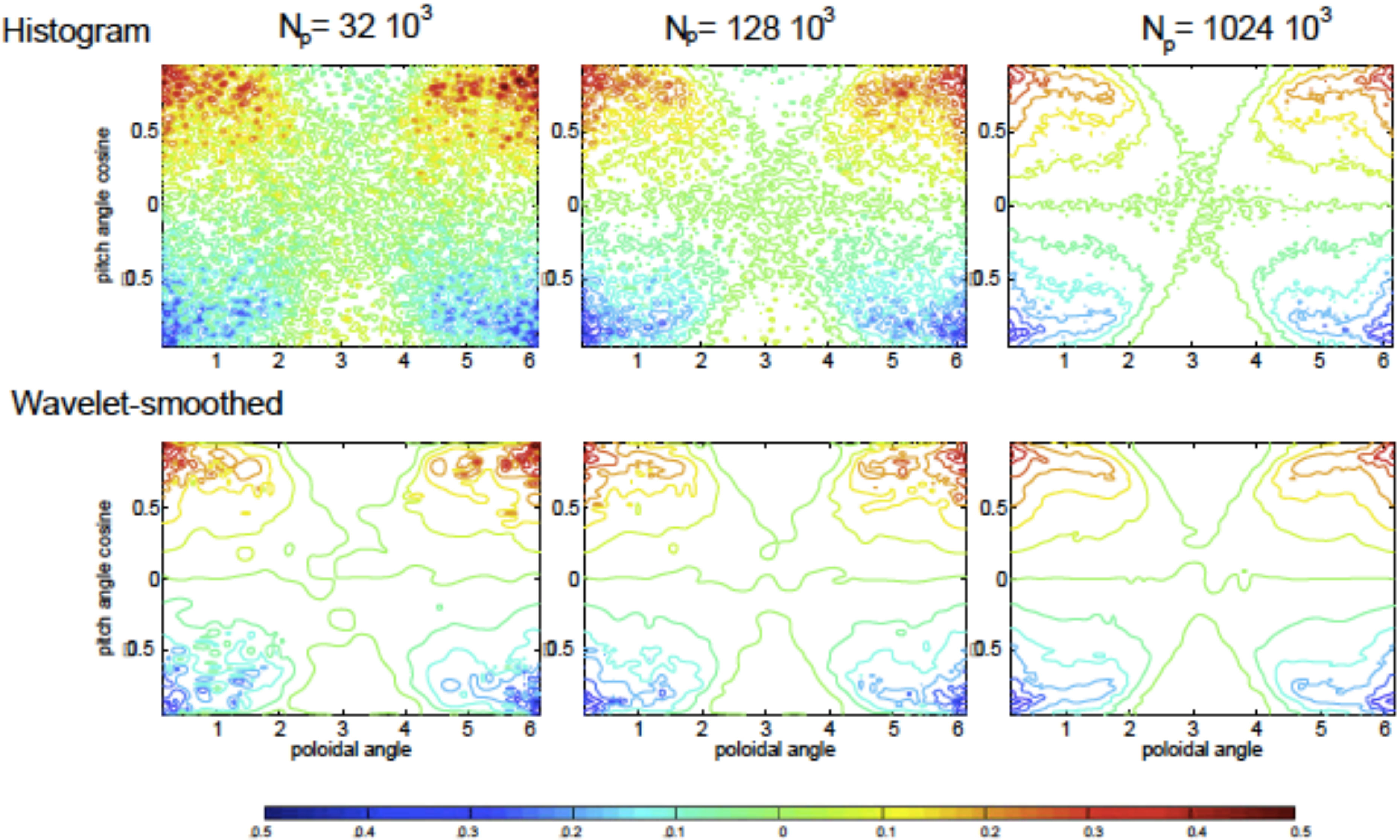
Noise reduction in plasma simulations using particles

- Accuracy of particle simulations is limited by noise (statistical sampling, not enough particles and grid effects)
- Wavelet based density estimation, accurate estimation of distribution functions with localized sharp features
- Preservation of moments in the distribution functions
- No a priori selection of a global smoothing scale
- No constraints on the dimensionality
- Computationally efficient: same order as for finite size particle approach

Nguyen van yen, del-Castillo-Negrete, Schneider, Farge and Chen, J. Comput. Phys., ***229****, 2010*

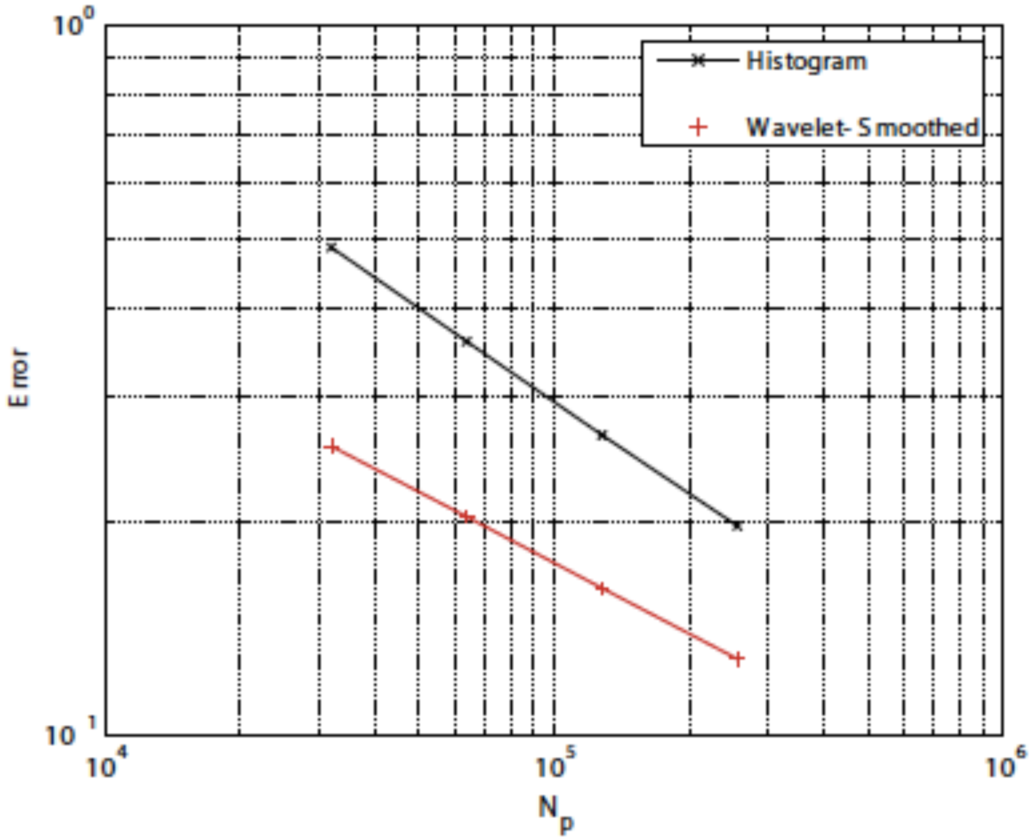
Noise reduction in plasma simulations using particles

Collisional guiding center transport data (Delta5d)



Noise reduction in plasma simulations using particles

Collisional guiding center transport data (Delta5d)



RMS error estimate with respect to the reference density computed with $N_p = 1024 \times 10^3$.

Error reduction by about a factor 2.

Nguyen van yen, del-Castillo-Negrete, Schneider, Farge and Chen, J. Comput. Phys., ***229****, 2010*

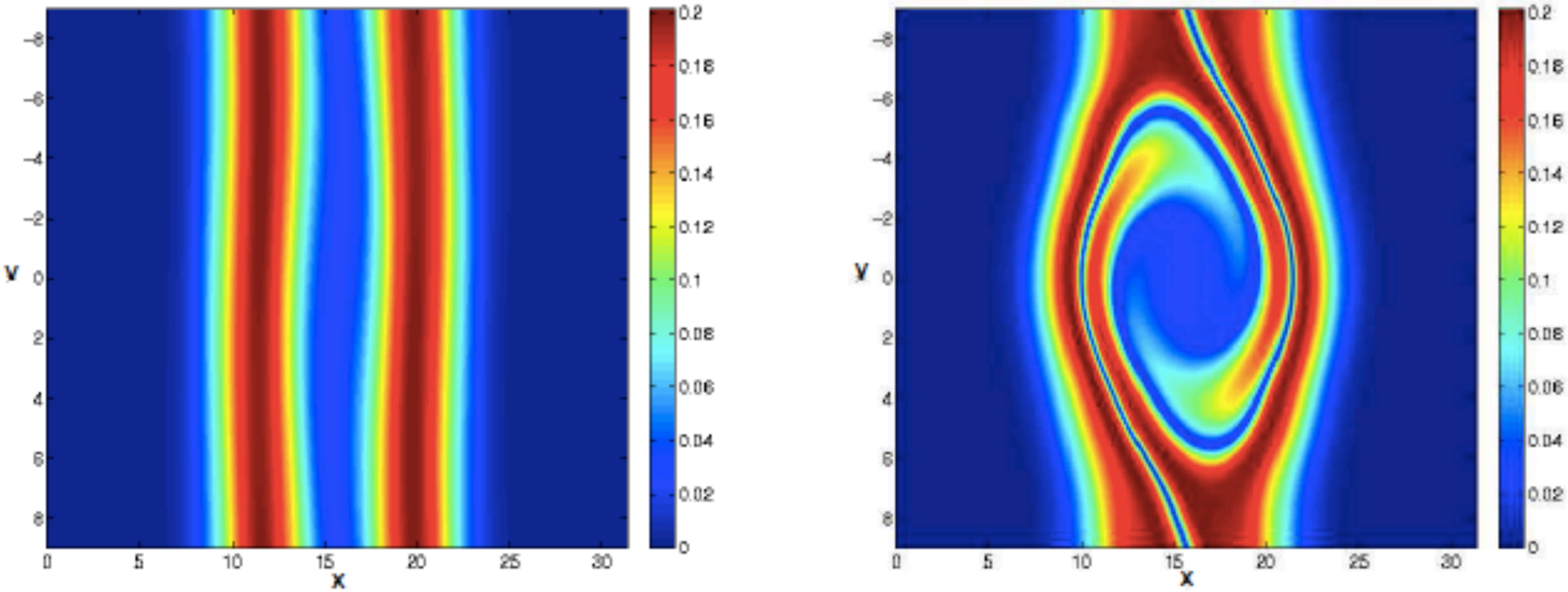
Particle in wavelets scheme for Vlasov-Poisson equation

- Plasma distribution function is discretized using tracer particles
- The charge distribution is reconstructed using wavelet based density estimation
- Wavelet expansion of the Dirac delta functions corresponding to each particle
- Wavelet Galerkin Poisson solver to compute the electric potential from the electron charge density (diagonal preconditioning)
- Improvement of precision compared to a classical PIC scheme for a given number of particles

Nguyen van yen, Sonnendrücker, Schneider and Farge, ESAIM Proc., ***32****, 2011*

Particle in wavelets scheme for Vlasov-Poisson equation

Two-stream instability test case

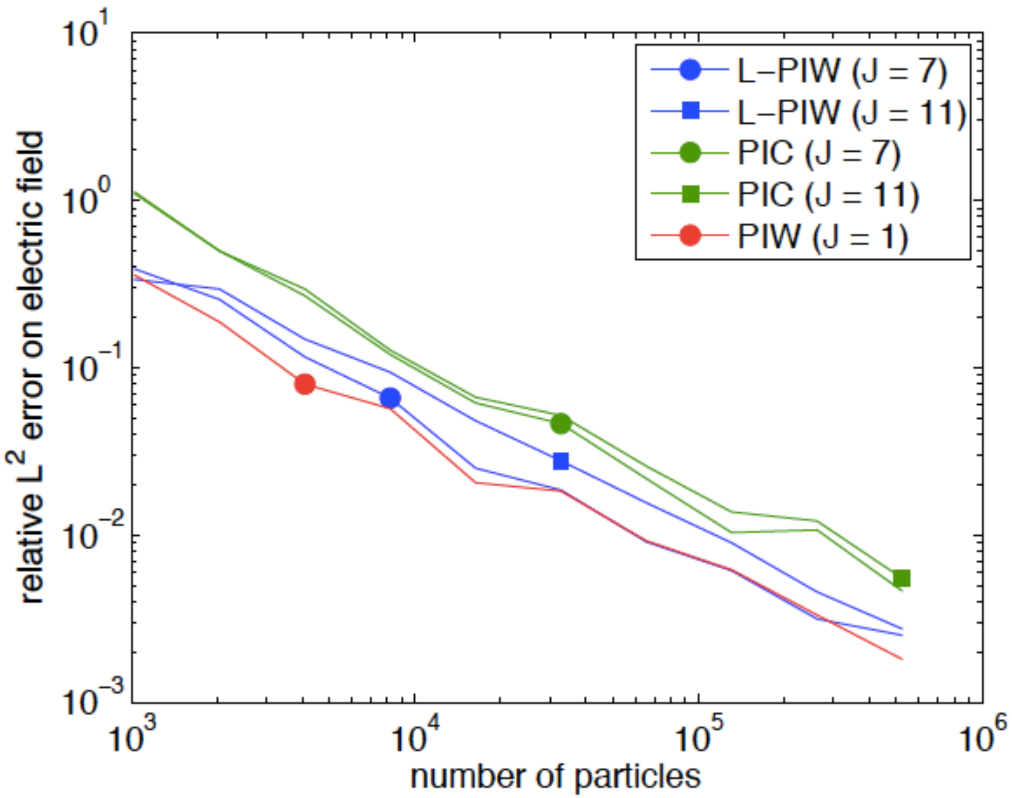


Particle distribution function at t=10 (left) and t=30 (right).

Nguyen van yen, Sonnendrücker,
Schneider and Farge,
ESAIM Proc., ***32****, 2011*

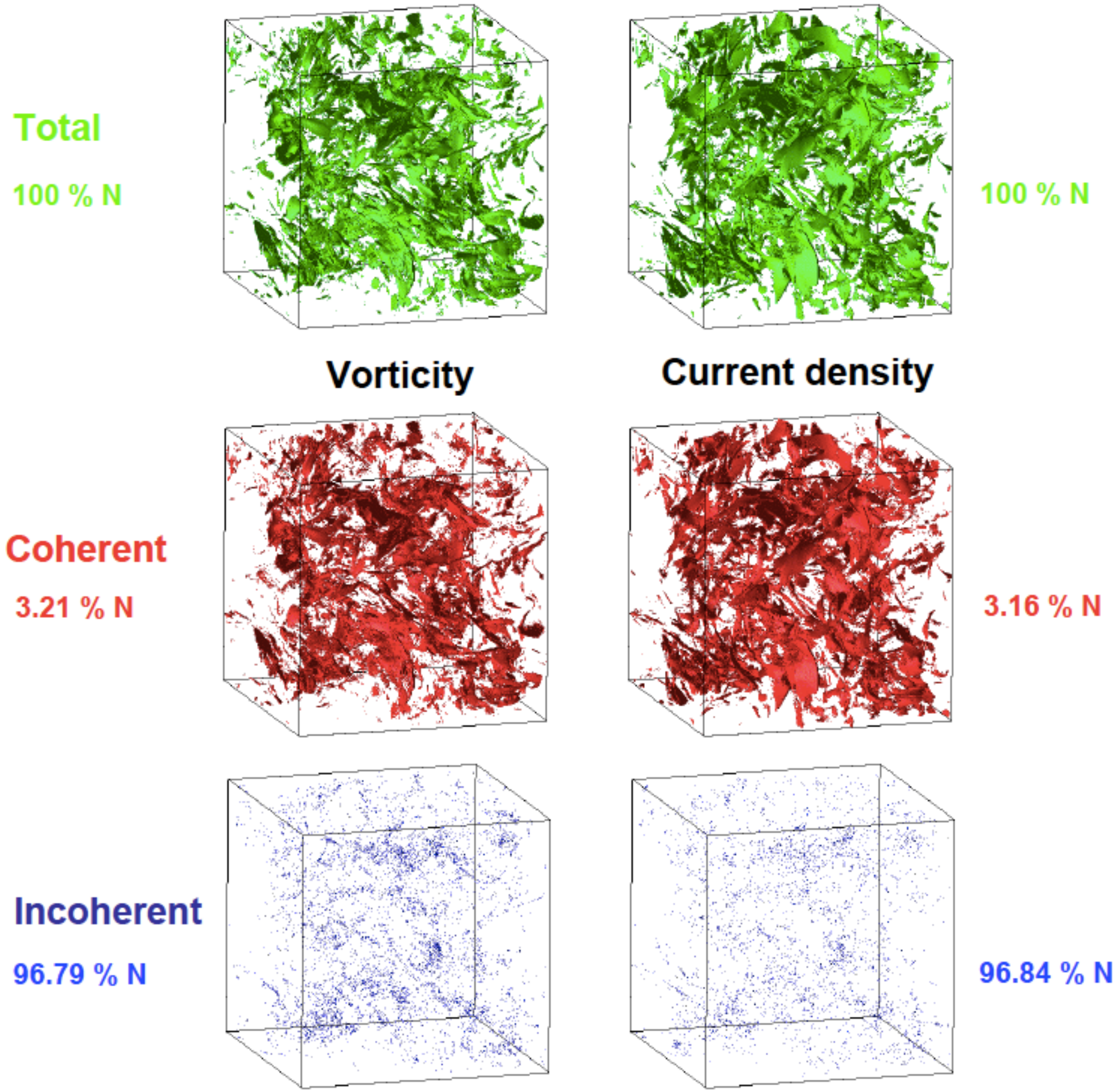
Particle in wavelets scheme for Vlasov-Poisson equation

L^2 error on the electric field at $t = 30$, as a function of number of particles



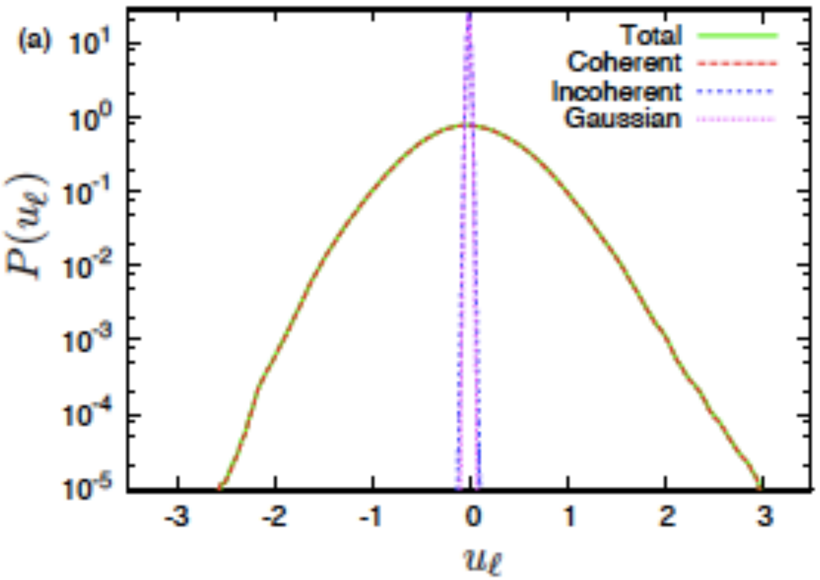
Nguyen van yen, Sonnendrücker, Schneider and Farge, ESAIM Proc., ***32****, 2011*

Coherent structures extraction in 3D MHD flow

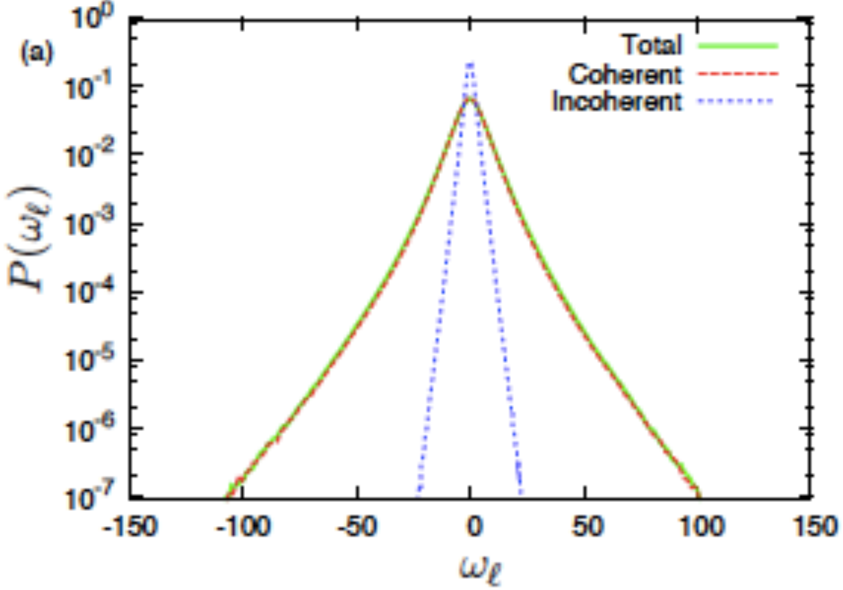


Yoshimatsu, Kondo, Schneider, Okamoto, Hagiwara & Farge Phys. Plasmas, ***16****, 2009*

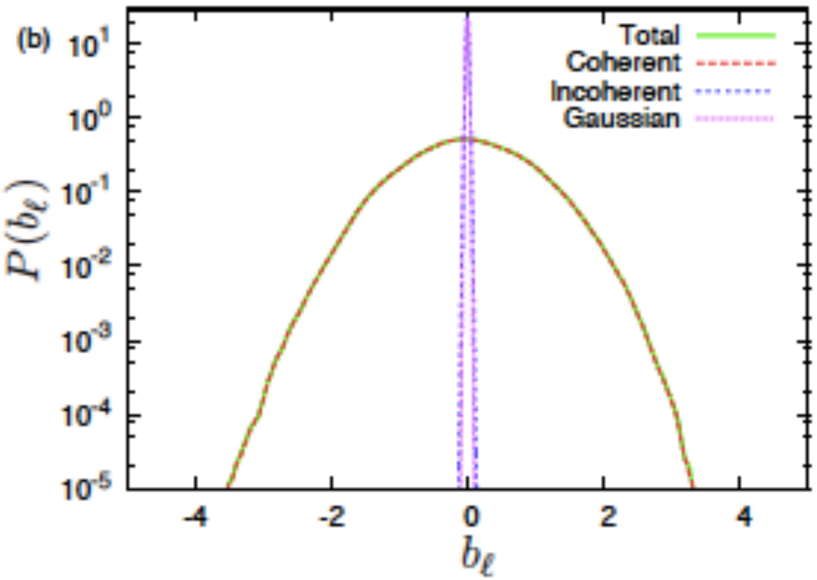
Velocity



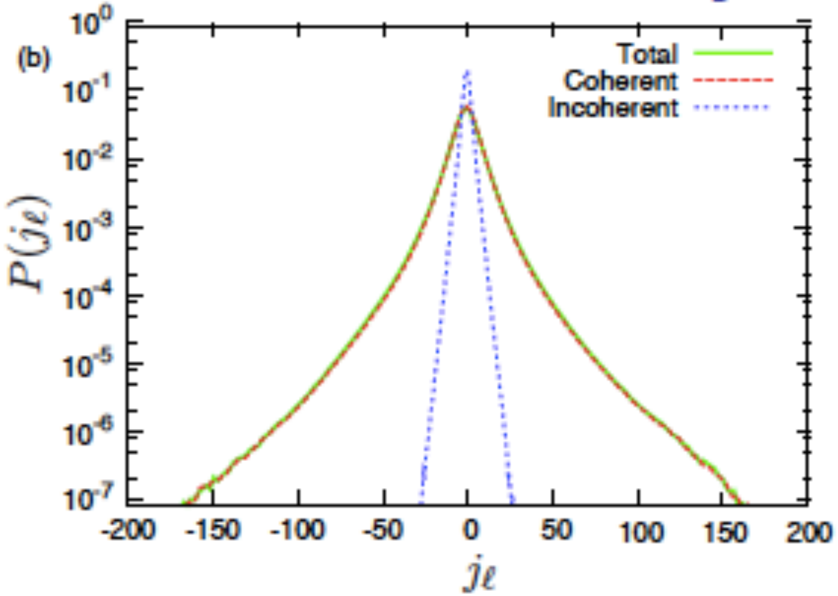
Vorticity



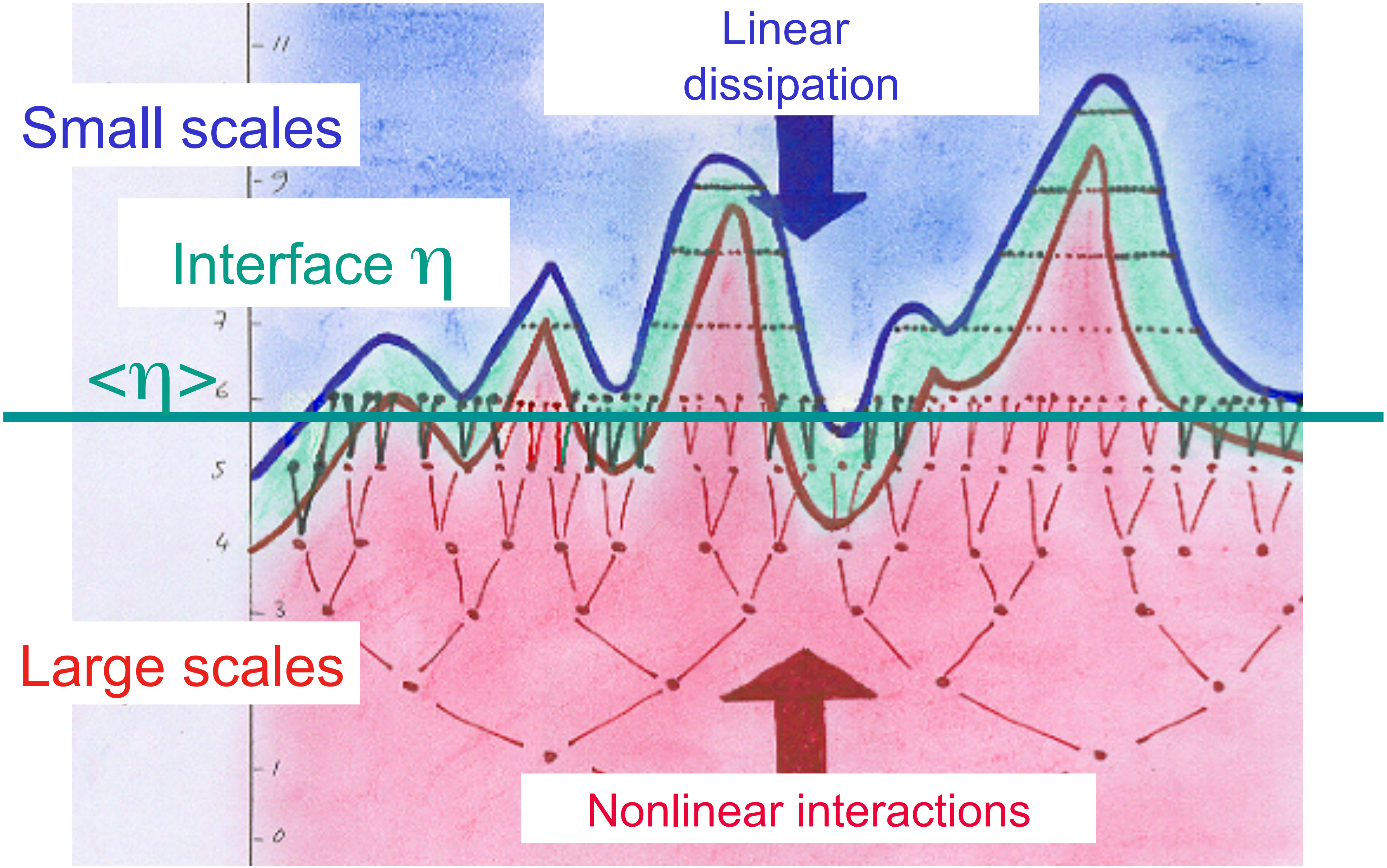
Magnetic field



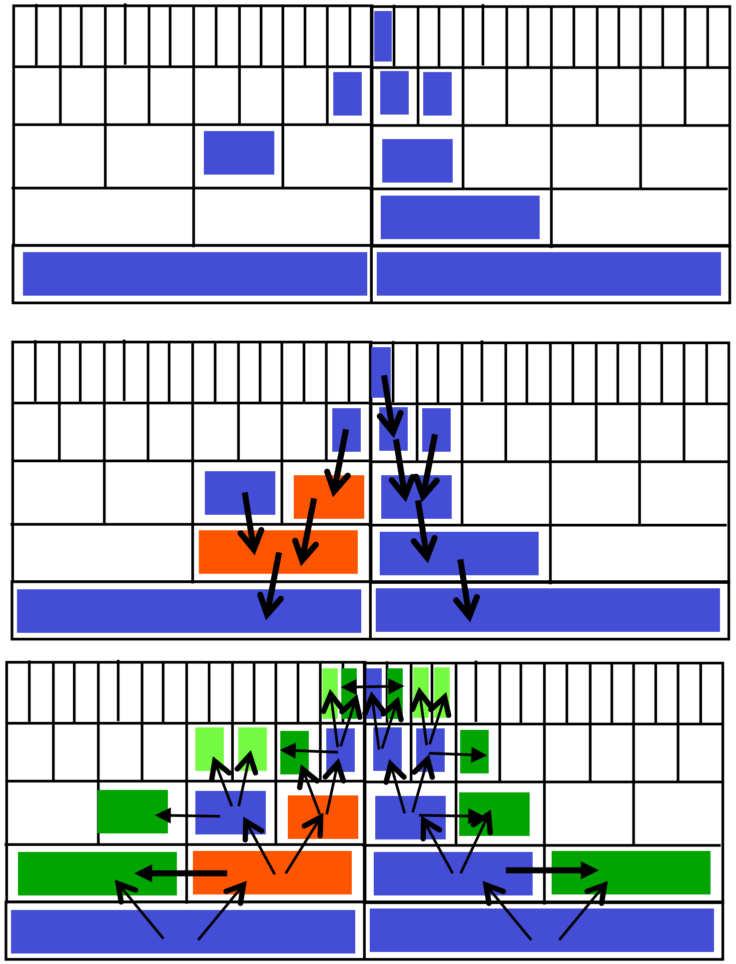
Current density



Coherent Vorticity Simulation (CVS)



Coherent Vortex Simulation (CVS)



Schneider & Farge, 2000,
Comp. Rend. Acad. Sci. Paris, 328

1. Selection of the wavelet coefficients whose modulus is larger than the threshold.

2. Construction of a ‘graded-tree’ which defines the ‘interface’ between the coherent and incoherent wavelet coefficients.

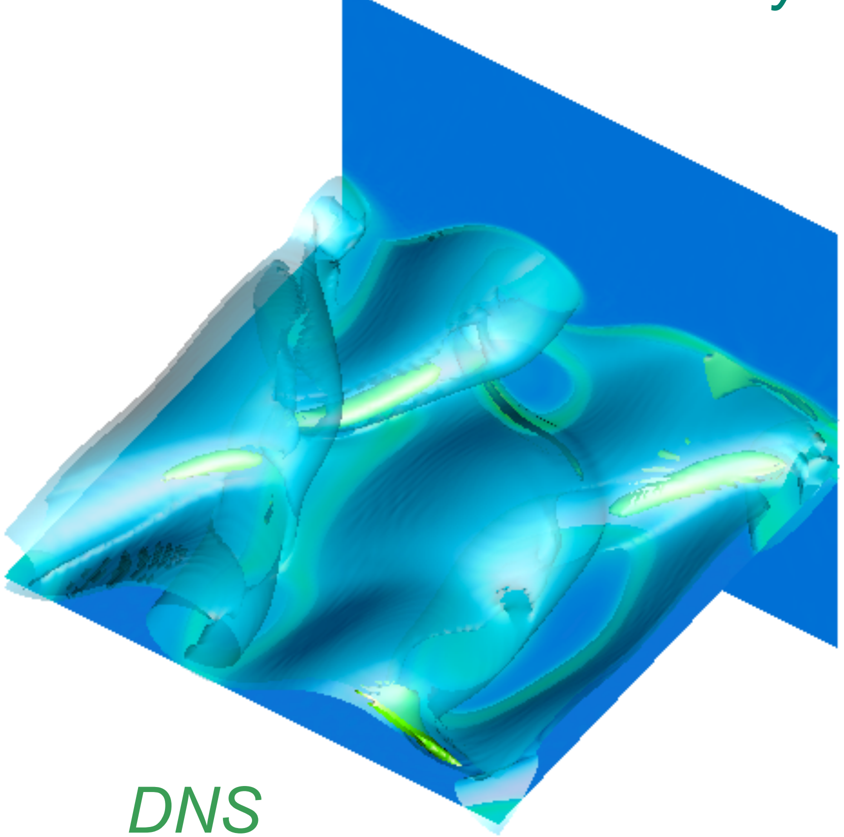
3. Addition of a ‘security zone’ which corresponds to dealiasing.

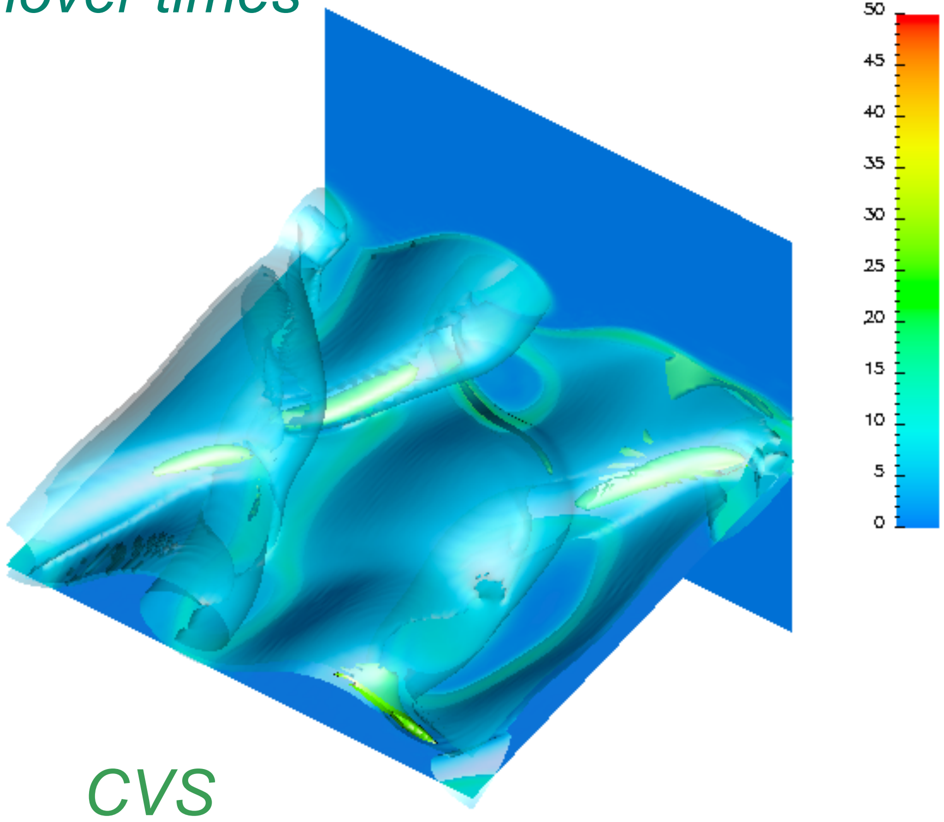
Schneider & Farge, 2002,
Appl. Comput. Harmonic Anal., 12

Schneider, Farge et al., 2005,
J. Fluid Mech., 534(5)

3D turbulent mixing layer

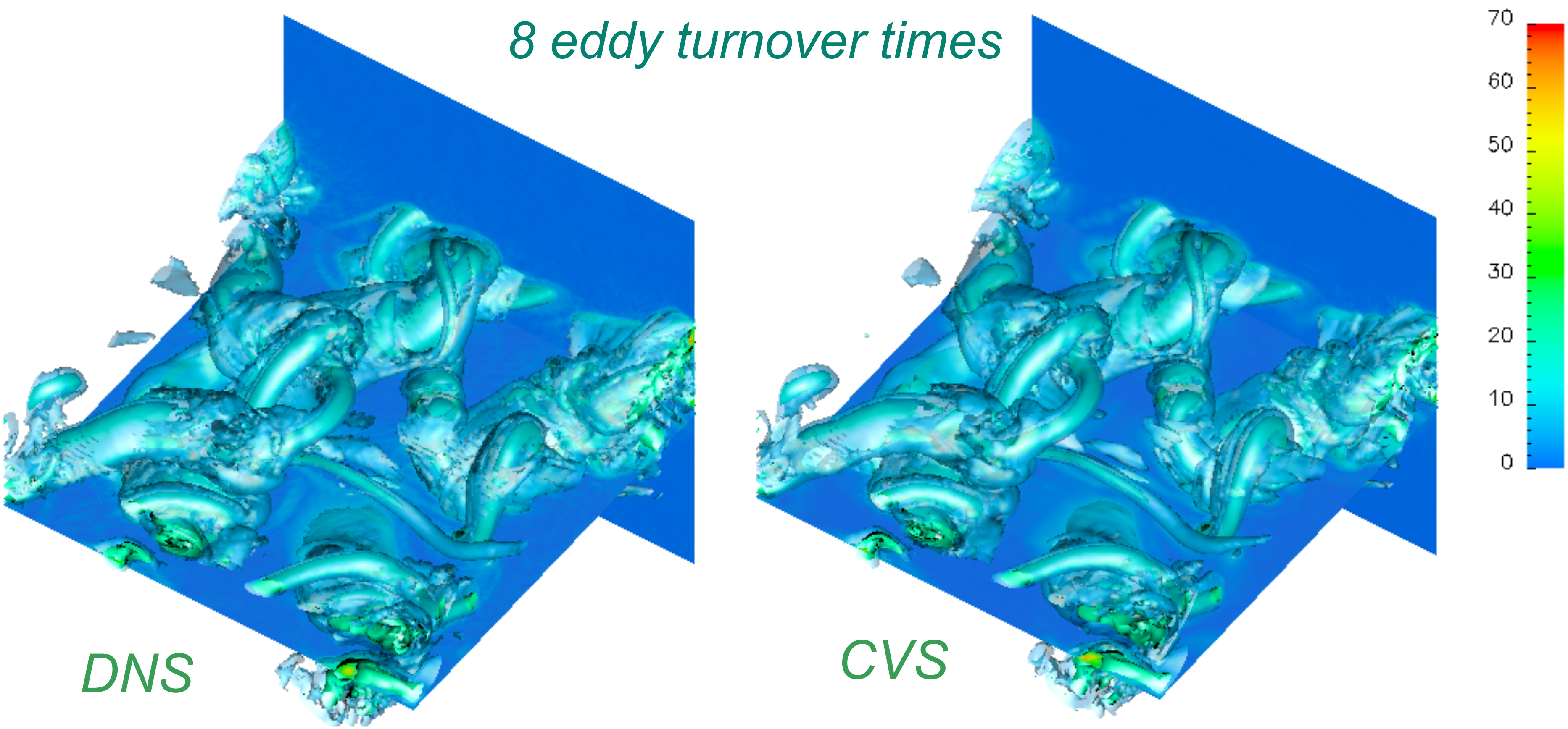
4 eddy turnover times





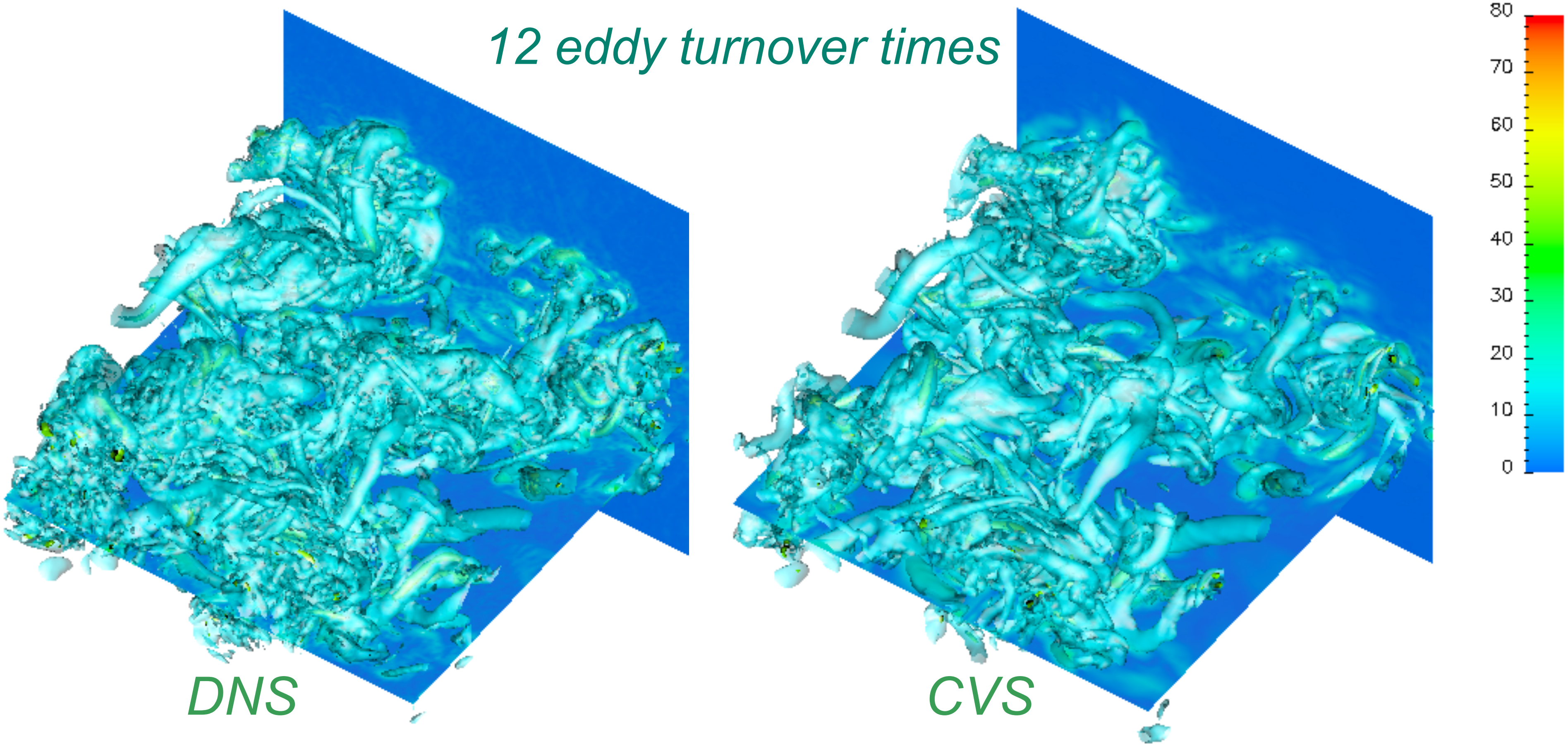
Schneider, Farge, Pellegrino, Rogers 2005, J. Fluid Mech., 534(5)

3D turbulent mixing layer



Schneider, Farge, Pellegrino, Rogers 2005, J. Fluid Mech., 534(5)

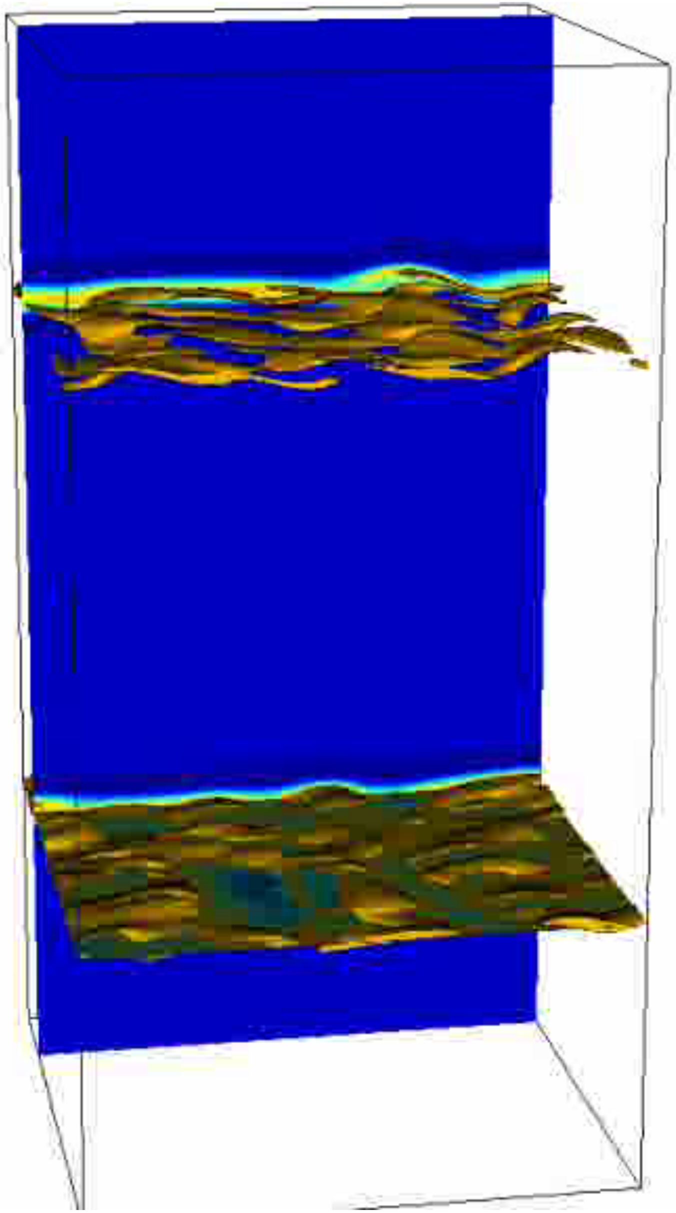
3D turbulent mixing layer

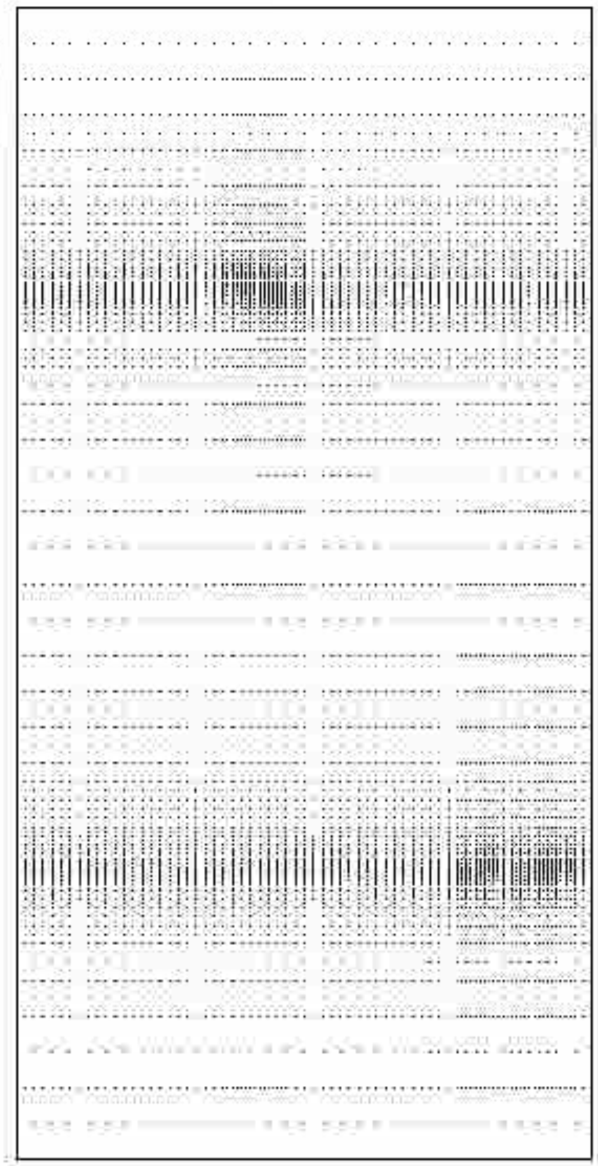


Schneider, Farge, Pellegrino, Rogers 2005, J. Fluid Mech., 534(5)

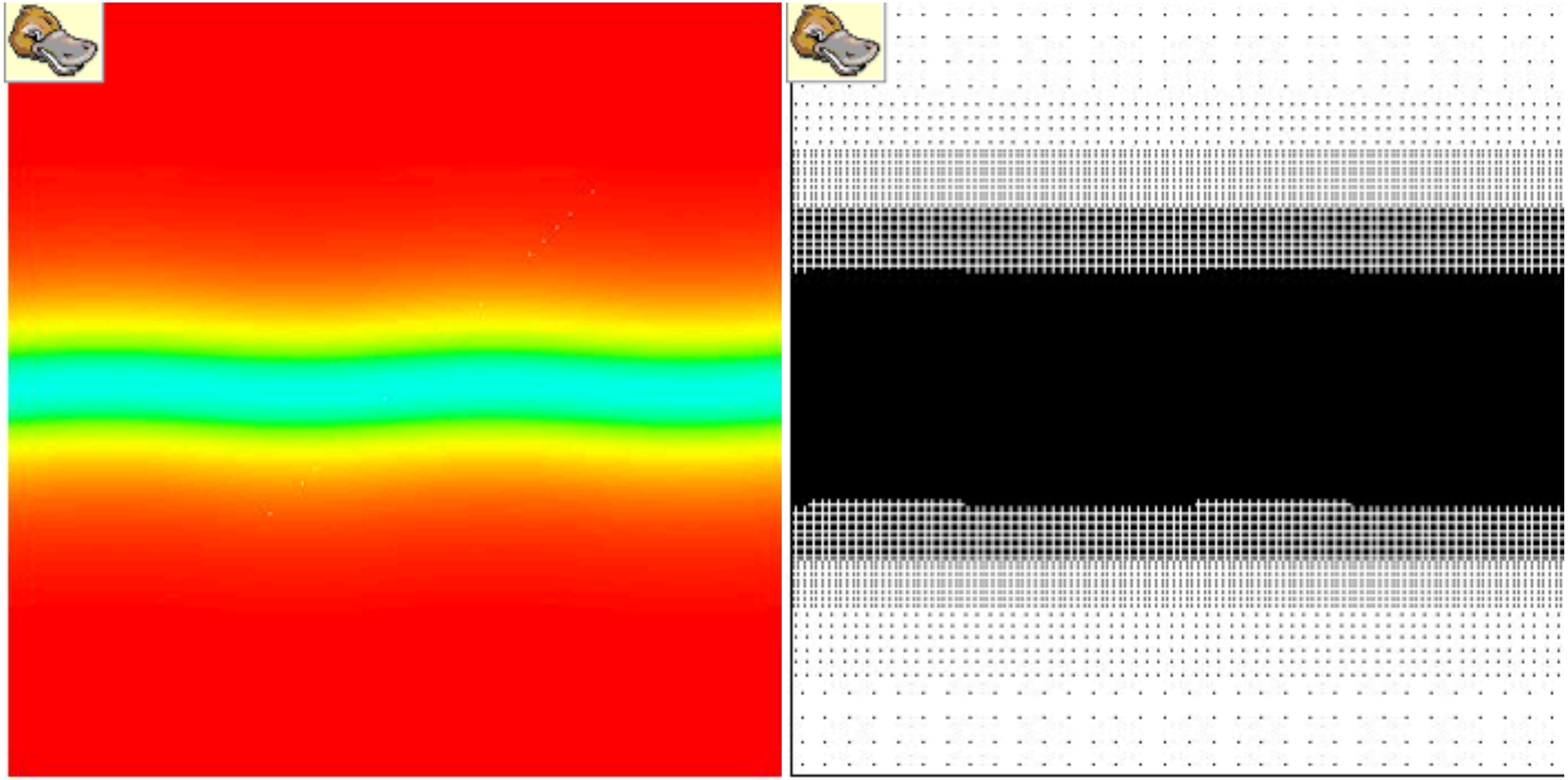
Adaptive computation using wavelets

Koster, Schneider, Griebel, Farge
Numerical Flow Simulation II,
***75**, Springer, 2001*





Adaptive computation using wavelets



Roussel and Schneider,
***75**, 2000*

Turbulence practice is the 'art of averaging'

Reynolds averaging (1883) :

Field f = *Mean* $\overline{f}$ + *Fluctuations* f'

with $\overline{f'} = 0 \qquad \overline{\overline{f}} = \overline{f}$

$$\overline{f+g} = \overline{f} + \overline{g} \qquad \overline{\partial f} = \partial \overline{f}$$

but nonlinearity is hard to handle since there is no scale separation :

$$\overline{fg} = \overline{f}\,\overline{g} + \overline{f'g'}$$

New way of averaging (1992):

$$f' = f'_c + f'_i$$

Fluctuations = *coherent fluctuations* + *incoherent fluctuations*

= *intermittent fluctuations* + *non-intermittent fluctuations*

Review papers on wavelets
http://wavelets.ens.fr

Marie Farge, 1992
Wavelet Transforms and Their Applications to Turbulence
*Ann. Rev. Fluid Mech., **24**, 395-457*

Marie Farge, Nicholas Kevlahan, Valerie Perrier and Eric Goirand, 1996
Wavelets and Turbulence
*IEEE Proceedings, **84**, 4, 1996, 639-669*

Marie Farge, Nicholas Kevlahan, Valérie Perrier and Kai Schneider, 1999
Turbulence Analysis, Modelling and Computing using Wavelets
Wavelets in Physics, ed. J. van den Berg, Cambridge University Press, 117-200

Marie Farge and Kai Schneider, 2002
Analyzing and computing turbulent flows using wavelets
Summer Course, Les Houches LXXIV, *New trends in turbulence, Springer*

Kai Schneider and Marie Farge, 2006
Wavelets: Mathematical Theory
Encyclopedia of Mathematical Physics, Elsevier, 426-437

Marie Farge and Kai Schneider, 2006
Wavelets: Application to Turbulence
Encyclopedia of Mathematical Physics, Elsevier, 408-419

Papers on applications to tokamaks
http://wavelets.ens.fr

Marie Farge, Kai Schneider and Pascal Devynck, 2006
Extraction of coherent bursts in turbulent edge plasma using orthogonal wavelets
*Physics of Plasmas, **13**(2), 042304, 1-11*

Romain Nguyen van yen, Diego del Castilo–Negrete, Kai Schneider, Marie Farge and Guangye Chen, 2010
Wavelet–based density estimation for noise reduction in plasma simulation using particles
*J. Comput. Phys., **229**(8), 2821-2839*

Romain Nguyen van yen, Eric Sonnendrücker, Kai Schneider and Marie Farge, 2011
Particle-in-wavelet scheme for the 1D Vlasov-Poisson equations
*ESAIM Proc., **32**, 134-148*

Romain Nguyen van yen, Nicolas Fedorczak, Frédéric Brochard, Kai Schneider, Marie Farge and Pascale Monier-Garbet, 2012
Tomographic reconstruction of tokamak edge turbulence light emission from single image using wavelet-vaguelette decomposition
*Nuclear Fusion, IAEA (International Atomic Energy Agency), **52**, 013005, 1-11*

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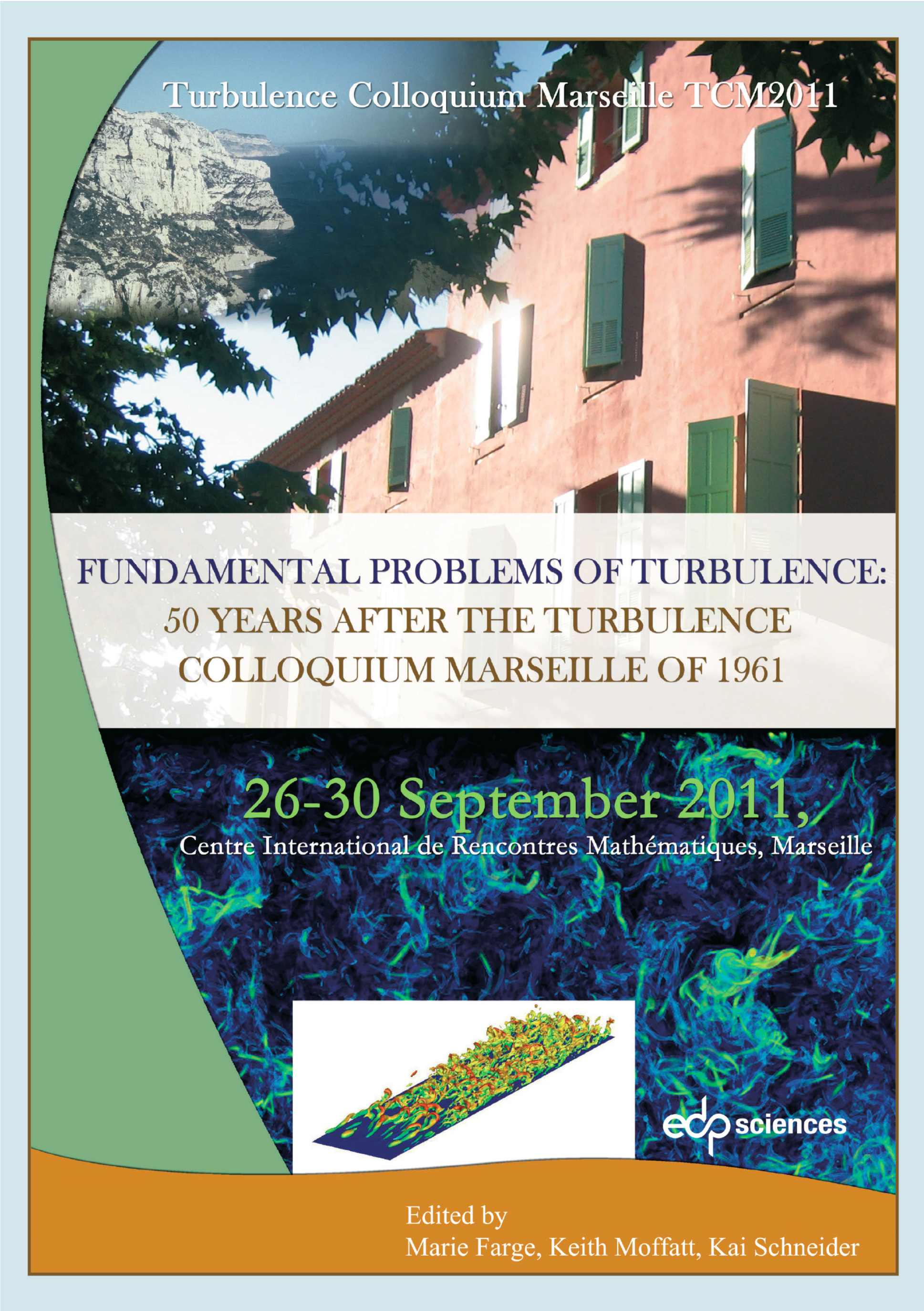

Summary

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Edited by Marie Farge, Keith Moffatt and Kai Schneider

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Marie Farge is CNRS Research Director at École Normale Supérieure, Paris, France.

Keith Moffatt is Professor Emeritus of Mathematical Physics at the University of Cambridge, and Fellow of Trinity College, Cambridge, United Kingdom.

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